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OUTLINES

OF

NATURAL PHILOSOPHY:

BEING THE

Heads of a Course of Lectures,

DELIVERED IN

COLUMBIA COLLEGE, NEW-YORK,

BY

JAMES RENWICK, A. M.

PROFESSOR OF NATURAL AND EXPERIMENTAL PHILOSOPHY, AND
CHEMISTRY, IN COLUMBIA COLLEGE.

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OUTLINES

OF

NATURAL PHILOSOPHY.

DEFINITIONS AND INTRODUCTORY REMARKS.

1. Philosophy is the science which inquires into the laws that regulate the phenomena of nature, whether in the intellectual or moral world.

The term Philosophy was first used by Pythagoras. CICERO, *Tusculanæ Quæstiones*, lib. 5. cap. 3.

2. Philosophy is divided into two branches, corresponding to the two great classes of substances; material and immaterial.

The material world is the province of Natural Philosophy; the properties and action of the bodies which compose the universe, the objects of its investigations.

3. The essential properties of matter are Extension, Mobility, and Impenetrability.

Attraction is often classed among the properties of matter, particularly that species of attraction known by the name of Gravitation; but we can conceive matter to exist which neither gravitates nor is attracted in any manner by any other portion of matter, and shall yet possess extension, impenetrability, and mobility.

4. Body is a separate and determinate quantity of matter, contained under some known figure, or existing in some determinate mode.

The same matter, by a change in its mode of existence, may form many different bodies.

5. Extension in three dimensions being a property of matter, it is, mathematically speaking, infinitely divisible; for geometric and arithmetic magnitude are divisible *ad infinitum*.

6. The actual division of matter can be carried to an almost incredible extent, as may be shown: by the great ductility of metals; by the distance at which odours affect the olfactory nerves; by the minuteness of the animalculæ discovered through the aid of the microscope, each of which is an organized being; by the colours of chemical solutions; by the smallness of the particles of light.

Still it may reasonably be doubted whether matter be infinitely divisible; the more probable supposition is, that it may finally be resolved into particles perfectly hard and incapable of further division.

These particles are called atoms; the theory that holds their existence the atomic theory; and it seems to be supported by the strong and conclusive evidence of numerous well ascertained facts. This evidence is, however, rather the object of chemical than of physical investigation.

7. Bodies, being of that class of geometric figures called solids, are inclosed by one or more boundaries.

8. The boundaries of solids are surfaces; the boundaries of surfaces, lines; the terminations of lines, points.

Geometry has been defined by Professor Leslie to be that branch of Natural Philosophy which treats of the property of matter called extension. LESLIE's *Elements of Geometry.*

9. By the term Impenetrability of Matter, it is intended to express the fact that no two particles of matter can occupy the same portion of space at one and the same time.

In this sense the rarest fluid is equally impenetrable with the hardest solid. If matter were penetrable by matter, all the bodies of the universe might be united in any space, however small. MUSCHENBROCK, vol. 1, § 81.

10. The bodies that compose the universe, as regards inhabitants of our earth, are either Terrestrial or Celestial.

Terrestrial bodies are divided into three kingdoms; the Mineral, the Vegetable, and the Animal.

11. Bodies differ from each other in respect of the ease or difficulty with which the particles they are composed of may be separated. Those in which the particles can be moved among each other by the smallest effort, are called fluid: those, where they adhere more strongly, solid bodies.

12. Motion is continual and successive change of place.

No body possesses within itself the power of changing its state, whether of motion or of rest. It cannot lose motion in any direction without communicating an equal amount to other bodies in the same direction; neither can it acquire motion in any direction, without diminishing the motion of other bodies by an equal quantity in that same direction.

This is the annunciation of that principle, called by some authors Inertia, and classed by them as one of the properties of matter. PLAYFAIR'S *Outlines*, § 21.

13. The foundation of the principles of Natural Philosophy rests upon experiment, and a careful observation of facts. The branch of knowledge that arranges and classifies facts, is called Natural History. BACON, *Novum Organum.*

14. When from a comparison of a number of facts, known, from experiment or observation, to be true, the existence of a more general fact is inferred, the inference is said to be made by *Induction.* Playfair, § 3.

> It is from induction that all certain knowledge of the laws of nature is derived. This method was first explained, and the rules for pursuing it laid down, by Lord Bacon, in his *Novum Organum.* Playfair's *Dissertation, Supplement Encyclopedia Britannica.*

15. When general principles have been once established by induction, we can often, by the application of mathematical reasoning, deduce from them conclusions as clear and certain as the principles themselves. Playfair, § 4.

16. We are said to explain a phenomenon when we show it to be necessarily included in some phenomenon or fact already known, or supposed to be known; and we consider one phenomenon as the cause of another, when we conceive the existence of the latter to depend on some force or power residing in the former. Playfair, § 6.

17. A fact assumed in order to explain appearances, and which has no other evidence of its reality but the explanation it is supposed to afford, is called an *Hypothesis.*

18. An explanation of any system of appearances or events, founded upon facts known to exist from evidence, independent of the facts themselves, is called a *Theory.* Playfair, § 8.

19. When one system of events or appearances is similar to another; and when we infer that the causes in the two systems are also similar, we are said to reason from *Analogy.* Playfair, § 9.

20. A theory discovered from induction, may be employed in

the discovery of new facts, and to predict the result of new combinations. PLAYFAIR, § 10.

The order of proceeding is in this case reversed, and is called the method of *Synthesis*, while the inductive method is called *Analysis*.

21. The rules for philosophizing are as follows, viz.:

(1.) More causes of natural things are not to be assigned than are both true and sufficient to account for the phenomena.

(2.) Of natural effects of the same kind, the same causes are, therefore, to be assigned as far as can be done.

(3.) The qualities of bodies which cannot be increased or diminished, and which are found common to all bodies on which experiments can be made, are to be considered qualities of all bodies whatsoever.

(4.) In Experimental Philosophy, propositions collected from the phenomena by induction, are to be considered as either entirely or nearly true, in spite of contrary hypotheses, until other phenomena occur, by which their accuracy may be more fully proved, or by which they may be rendered liable to exceptions. NEWTON's *Principia*. Vol. 3. p. 2. of Horsley's edition.

The principle of the sufficient reason is also often of great use in philosophizing; it may be announced as follows, viz. Nothing exists in any state that is not determined by some *reason* to be in that state, rather than in any other. PLAYFAIR, § 12.

22. Experiment is not only valuable as the foundation of all Natural Philosophy, but is indispensable in a course of lectures upon that subject. It is not however to be expected that all the experiments made by the founders of the science, can be performed before a class. It is therefore proper to explain the purposes for which experiment is introduced.

It verifies the results of our reasoning, and shows if all the circumstances have been taken into account:

It exemplifies how general principles may be applied to explain particular facts:

It impresses the mind more forcibly with the truth of the principles that have been laid down; and of the inferences deduced from them. PLAYFAIR, § 13.

The definition of motion, which has been given above, involves the consideration of space and time:

23. Space is either absolute or relative. Absolute space is extension without limit, immoveable, but penetrable by matter. Relative space is that part of absolute space which our senses define, by its relation to bodies within it.

24. Place is also either absolute or relative. Absolute Place is the portion of absolute space occupied by a body. Relative Place is the space a body occupies, considered with relation to other bodies.

25. Absolute Time is an abstract idea, of which no satisfactory definition can be given.

Relative time is a portion of duration measured by means of motion.

26. The cause which puts a body in motion, whatever be its nature, is called a Force.

The direction of a force is the straight line in which it tends to cause the point to which it is applied to move.

When several forces are applied to the same body at the same instant of time, they reciprocally modify each other. If they entirely destroy each other, so that the body is kept at rest by their joint action, we say that an equilibrium takes place among them, or that the body is in equilibrio. Mathematicians have succeeded in reducing all considerations of motion to mere questions of equilibrium. Poisson, *Mechanique*, § 2.

27. Mechanics is the branch of Natural Philosophy which treats of the motion and equilibrium of bodies. It also treats of the construction of Machines. The first of these is called by Newton, Rational; the second, Practical Mechanics.

Rational mechanics is divided into two parts; Statics, which treats of equilibrium, and Dynamics, which treats of the motion of bodies.

The first principles of mechanics apply equally to solid and to fluid bodies, but are modified by the individual nature of each. Fluid bodies, however, present peculiar difficulties to the investigation of their mechanical action. We consequently treat of the statics and dynamics of fluids separately, calling the parts of the course devoted to them Hydrostatics and Hydrodynamics.

The action of bodies upon each other necessarily involves motion. Mechanics, in its most extended sense, would therefore include every department of Natural Philosophy, together with Chemistry. We do not, however, carry the definition to this extreme.

28. There is a class of material substances, whose distinctive property is radiation from a centre. The consideration of the action of these bodies, forms a separate department, to which the name of Physics is now restricted.

The radiant substances known, are Heat, Light, Electricity, and Magnetism. Philosophers still dispute whether all these are not modifications of some one principle; and there is with some a strong doubt whether they are material substances or not.

29. Neither Mechanical nor Physical Action, in the restricted sense, produces any change in the internal constitution of bodies. When, however, we place certain bodies in close contact with each other, an action takes place that changes their

nature, character and properties; their mode and state of existence. This action is called Chemical, and the science which considers it, Chemistry.

30. Mechanics and Physics are still farther abridged by the separation of Astronomy from them. It is the science which treats of the motion and appearances of the heavenly bodies.

This division of Natural Philosophy into three great branches, Mechanics, Physics, and Astronomy, (and Chemistry might form a fourth,) affords the means of ranking the objects of study in regular order, and introduces the advantages of philosophical arrangement in the classification and comparison of facts. No one of these departments, however, is entirely distinct and separate from the other. Physical considerations continually arise in treating of mechanical science; mechanical reasoning illustrates physical facts; while the theory of astronomy would fall to the ground, were it not for the powerful aid furnished by mechanical philosophy.

31. The study of Natural Philosophy is attended with many advantages.

It has a tendency to add to the conveniences and comforts of social life; to improve the state of agriculture, commerce, and manufactures; to embellish all the fine and useful arts; to lead us to know and prove the existence of a provident deity; and to enable us to understand his attributes and admire his wisdom.

MECHANICS.

SECTION FIRST.—STATICS.

OF FORCES.

The several circumstances which should be known with regard to a force, are, its point of application, its intensity, and its direction.

When a point is in motion, it will be known by the fact that its perpendicular distance from one or more of three planes mutually intersecting each other in space, is continually changing, wherefore:

32. To determine the position in space of the point of application of a force, we refer it to three planes taken at will perpendicular to each other. The perpendicular distances from the point to these three planes are called Co-ordinates.

The mutual intersections of the planes are called the axes of these co-ordinates, and the axes and co-ordinates are parallel to each other.

The common intersection of the three planes is called the origin of the co-ordinates. (*Leçons des Ecoles Polytechniques.*) Vol. 5.

33. Forces are made commensurable quantities by referring them to some conventional force as the unit; they may then be represented either by lines or numbers.

The intensity of a force is usually represented by a line drawn in the direction of the force from its point of application, and this is the mode of construction adopted by the best writers on statics. POISSON, § 4.

34. The direction of a force in space is determined by a knowledge of the angles it makes with three lines drawn through its point of application parallel to the axes of the co-ordinates. Poisson, §6.

If we call these three angles a, b, and c, we shall find a constant relation to exist among them determined by the equation. $\text{Cos.}^2 a + \text{cos.}^2 b + \text{cos.}^2 c = 1.$ Poisson, § 8.

If all the forces are in one plane, the equation becomes $\text{Cos.}^2 a + \text{cos.}^2 b = 1.$ Poisson, § 9.

If they are parallel, $\text{Cos.}^2 a = 1.$ and $a = 0°$ or $a = 180°$. Poisson, § 10.

35. When the direction of a force is once determined, its action will not be changed by transferring the point of application to any other point in its direction, provided we consider this second point as attached to the other by an inflexible straight line, and that the intensity and direction of the force remain the same. Poisson, § 28.

COMPOSITION AND RESOLUTION OF FORCES.

36. When a body is submitted to the simultaneous action of several forces that act upon it in different directions, but are not in equilibrio, it is evident that it must move in a determinate direction, and that there is no impropriety in attributing its motion to a single force acting in that direction. This force is called the Resultant of those that have caused the motion, and they are called Components.

This resultant identically replaces its components, and is in consequence in equilibrio with them when applied to a given point in a direction directly contrary to its true one. Poisson, § 11.

37. The resultant of two forces is represented in magnitude

and direction by the diagonal of a parallelogram whose sides represent the magnitude and direction of these two forces. Poisson, *Additions.* Poisson, § 13, 14, 15, and 16. *Mechanique Celeste*, vol. 1, § 1.

By means of this theorem, every question with regard to the composition of two forces into a single one, and the decomposition of a single one into two others, is reduced to the resolution of a plane triangle.

For instance : if two forces be represented in magnitude by P and Q, the angle contained between their directions by *m*, the value of the resultant R may be found by the equation.

$$R = P^2 + 2\,P\,Q.\,\cos.\,m. + Q^2.$$

That of the angle which R makes with P by the equation.

$$\text{Sin } x = \frac{Q.\,\sin.\,m.}{R}$$

Poisson, § 17.

38. When three forces are in equilibrio round a point, each of them may be represented by the sine of the angle contained by the directions of the other two.

39. When three forces are in equilibrio round a point, they must be all in the same plane. Poisson, § 18.

40. If three forces applied to the same point are represented in magnitude and direction by the three sides of a triangle, they are in equilibrio round it.

The converse of this is also true ; therefore, if three forces be in equilibrio, any two of them are greater than the third. Gregory, § 46, 47.

41. To find the resultant of any number of forces applied to the same point, whether they be in the same plane or not: the resultant of two of them must be first taken, and this resultant combined with a third force; the second resultant with a fourth force,

and so on, until all the forces have been employed; the last resultant will be that of all the forces.

> This rule gives rise to a remarkable geometrical construction; for if we form a portion of a polygon by lines equal and parallel to those which represent a number of forces applied to one point, the resultant of the whole will be represented by the line which joins the extremity of the last line to the point where the construction was commenced, and completes the polygon. POISSON, § 19.

42. If three forces act at right angles to each other, their resultant will be represented by the diagonal of a rectangular parallelopiped whose sides represent in magnitude and direction the three components.

> If we call the resultant R, and the three components X, Y, and Z. $R = X^2 + Y^2 + Z^2$, and if the three angles they form be called a, b, c, $X = R. \cos. a$, $Y = R \cos. b$, $Z = R. \cos. c$. POISSON, § 20.

43. If any number of forces F F′ F″, &c. be applied to the same point, and if we represent by a, b, and c, the angles which the force F makes with three rectangular axes; by a', b', and c', the angles the force F′ makes with the same axis, and so on; the whole of these forces may be replaced by three others, X, Y, and Z, parallel to these axes by means of the following equations.

$X = F. \cos. a + F.' \cos. a' + F.'' \cos. a'' +$ &c.

$Y = F. \cos. b + F.' \cos. b' + F.'' \cos. b'' +$ &c.

$Z = F. \cos. c + F.' \cos. c' + F.'' \cos. c'' +$ &c. POISSON, § 21.

44. If the whole of these forces are in equilibrio, the equations become:

$F. \cos. a + F.' \cos. a' + F'' \cos. a'' = 0.$

$F. \cos. b + F.' \cos. b' + F'' \cos. b'' = 0.$ POISSON, § 22.

If the point of application of the forces F F′ F″ rests upon a given surface, it is no longer necessary that the resultant should = 0, in order to produce equilibrium; it will be sufficient that it be a normal to the surface. This normal force will be destroyed by the resistance of the surface, which may in consequence be considered as a force equal and opposite to it. If, therefore, we conceive the surface to be removed, and the point to be situated in free space, the conditions of equilibrium will exist, provided we substitute for the action of the surface a force equal and directly opposite to the resultant of the others.

Poisson, § 39.

OF PARALLEL FORCES.

45. The resultant of two parallel forces acting in the same direction, divides the line of application into parts reciprocally proportionable to the component, is parallel to them, and equal to their sum. Poisson, § 31.

If the points of application and the intensity of the parallel forces remain the same, the resultant will pass through the same point, whatever be the direction of the forces, so that if the forces turn round their respective points of application, the resultant will also revolve around its point of application.

The momentum of a force with respect to any plane, is the product of the force into the perpendicular let fall from its point of application upon the plane. Poisson, § 39.

The momentum of a force with regard to a point, is the product of the force into the perpendicular let fall from the point upon the direction of the force. Poisson, § 52.

46. The momentum of the resultant of any number of parallel forces with respect to a plane, is equal to the sum of the momenta of these forces with respect to the same plane.

These momenta may be either positive or negative: they are positive when the force and the ordinate of its point of application have like signs; negative, when their signs are unlike. Poisson, § 39.

47. The Centre of Parallel forces is the point through which all the successive directions of the resultant pass, while the components turn round their respective points of application without ceasing to be parallel. Poisson, § 37.

It follows from this definition, that if a solid body is acted upon by any number of parallel forces, and if their centre be determined and supposed to be fixed, the body will remain in equilibrio around this point in every possible position, provided the forces remain parallel, and continue to act on the same points of the body.

48. A system of parallel forces is in equilibrio,

(1.) When the sum of the several forces = 0, and

(2.) When the sum of their momenta, in relation to two planes parallel to their direction, is also = 0. Poisson, § 42.

By the word sum we are to understand the aggregate of the momenta incorporated according to their signs, using the affirmative sign for the momenta of those powers that lie on one side of the point of application, and the negative for those that lie on the other. Gregory, *Mechanics*, § 60.

For the theorems by which the centre of parallel forces may be determined, see Poisson, § 39, 40.

CENTRE OF GRAVITY.

49. The force by the action of which bodies are precipitated to the surface of the earth, is called Gravity. Although the directions of this force in different places would converge towards

the centre of the Earth on account of its spheroidal form; yet, in consequence of the great size of the earth, when compared with the dimensions of the bodies upon its surface, we may, without any sensible error, suppose the action of gravity to be parallel to itself, throughout the whole extent of any one body.

Experience shows, as we shall see in another place, that the force of gravity is not the same on every part of the surface of the earth, yet this variation may also be neglected without any error, and we may consider a heavy body as a collection of a number of particles of matter to which are applied an equal number of equal and parallel forces acting in the direction that is pointed out by the plumb line. Poisson, § 92.

50. These forces have a resultant that is equal to their sum, and parallel to their direction, and this resultant forms what is called the weight of the body. Poisson, § 93.

51. Since every particle in a heavy body is solicited by a parallel force, it follows, that if it take successively any number of different positions, as regards the direction of these forces, their resultant will always pass through one particular point. This point, which, in general terms, we have called the Centre of Parallel forces, is, in this particular case, called the Centre of Gravity. Its characteristic property, in solid bodies, is, that if it be supported, the body to which it belongs will remain in equilibrio around it in every possible position; this follows from the fact that the resultant of the several forces which solicit the body, passes through the point that is thus supported. Poisson, § 95.

We also see that when a solid body is supported by a point that is not the centre of gravity, it will be in equilibrio when the line that joins the point that is supported to the centre of gravity is vertical, and only then. Poisson, § 95.

It is evident likewise that if a body be suspended in equilibrio by a thread, that this thread will be vertical, and the prolongation of its direction will pass through the centre of gravity. This affords a method of determining the centre of gravity of a solid body, however irregular or heterogeneous, by experiment. Poisson, § 96.

When a heavy body is at liberty to move round a fixed point, it will not be at rest until its centre of gravity is either in the highest or lowest possible position.

The first of these positions forms a tottering, the last a stable equilibrium.

A body cannot rest upon a base unless a perpendicular to the surface of the earth from its centre of gravity fall within the base. Playfair, § 108.

This perpendicular line is called the Line of Direction of the Centre of Gravity.

The motions of animals are regulated by the foregoing principles, and produced by the action of their muscles in throwing forward the line of direction of the centre of gravity. Borellus, *De motu animalium.*

The stability of the famous leaning tower of Pisa, as well as many other curious phenomena in mechanics, depends on the properties of the centre of gravity.

52. The position of the centre of gravity may be found by experiment, as we have previously seen: To find it by calculation, we must resolve the following problem.

To find the centre of gravity of a system of bodies whose individual centres of gravity are known.

If we call the co-ordinates of the principal centre of gravity, x, y, and z, the several bodies that compose the system A, B, C, D, &c. the co-ordinates of A, a, a', a'', of B, b, b', b'', &c. we shall have the following equations.

$$x = \frac{Aa + Bb + Cc + Dd, + \&c.}{A + B + C + D, + \&c.}$$

$$y = \frac{Aa' + Bb' + Cc' + Dd', + \&c.}{A + B + C + D, + \&c.}$$

$$z = \frac{Aa'' + Bb'' + Cc'' + Dd'', + \&c.}{A + B + C + D, + \&c.}$$

Poisson, § 99.

If the body be homogeneous, and referred to one plane, we shall have the following equation:

$$d = \frac{Aa + Bb + Cc + Dd, \&c.}{M}$$

d. being the distance of the centre of gravity from the plane, and M the mass of the body. Hence we have the following rule, that the whole volume of a body multiplied by the distance of its centre of gravity from a given plane, is equal to the sum of all its particles multiplied by their respective distances from the same plane.

Although geometric lines and surfaces have no actual weight, yet it often happens that it is necessary to determine the position of their centres of gravity. In order to do this, we consider all the points in them as loaded with equal weights, or acted upon by equal and parallel forces.

The centre of gravity of a triangle may, by geometric reasoning, be demonstrated to be in the line that joins its vertical angle to the middle of its base and at a distance from the vertex equal to two thirds of this line. The centre of gravity of a triangle may therefore be found by drawing lines from the two angles to points bisecting the opposite sides.

The centre of gravity of a triangle being thus found, it is easy to find that of any polygon by dividing it into triangles, and applying the above formulæ. Poisson, § 108.

A similar chain of reasoning will lead us to discover that the centre of gravity of a solid pyramid may be found by drawing a straight line from the vertex to the centre of gravity of the base, and dividing it into four equal parts; the centre of gravity is at the distance of three-fourths of this line from the vertex. As this applies to a pyramid of any number of sides, the construction may be extended to the case of a cone.

In general, however, the centre of gravity of a mathematical figure can only be found by the application of the fluxional calculus to the formulæ given above. The general method is to refer the figure to three rectangular co-ordinates. POISSON, § 101, 102.

The method of finding the centre of gravity of a plane curve is simplified by supposing one of the planes of the co-ordinates to correspond with that of the curve. POISSON, § 103.

The same formulæ may in like manner be applied to the surface of solids and to the solids themselves.

53. The solid formed by the revolution of a plane curve round a straight line within its plane, is equal to the product of the area of the generating curve into the length of the circular arc described by its centre of gravity in its revolution.

And the surface described by a plane curve revolving in a similar way is equal to the length of the generating curve multiplied into the length of the circular arc described by its centre of gravity.

This theorem was discovered by *Guldin*, the mathematician, whose name it bears. GREGORY, § 125. M'LAURIN, *Fluxions*, § 236.

The knowledge of the position of the centre of gravity is of great importance in almost every part of Mechanics. The following are some of the most useful cases.

The centre of gravity of a straight line bisects it.

The centre of gravity of the perimeter of a polygon may be found when the centre of gravity of a straight line is known by applying the formulæ, § 61.

The distance of the centre of gravity of an arc of a circle from the centre of the circle is a fourth proportional to the radius, the chord, and the arc. POISSON, § 104.

The centre of gravity of an arc of a cycloid, that is divided into two equal parts by the vertex, is in the diameter of the generating circle at one third of the perpendicular height of the arc from the vertex. POISSON, § 105.

The distance of the centre of gravity of a parabola from its vertex is equal to three fifths of the axis. GREGORY, § 119.

In a circular sector, the distance of its centre of gravity from the centre of the circle is a fourth proportional to two thirds of the radius, the chord, and the corresponding arc. POISSON, § 110.

The centres of gravity of the surfaces of cylinders, cones, and conic frustrums, are as far from the origin of the co-ordinates as the centres of gravity of the plane figures which are their vertical sections.

The centre of gravity of a hollow and a solid cone do not coincide, for in the one,

$$x = \frac{2a}{3}; \text{ in the other, } x = \frac{3a}{4}$$

The centre of gravity of the surface of a spheric segment bisects its versed sine.

In a conic frustrum, if R be the radius of the greater end, r that of the less, and h the altitude of the frustrum,

$$x = \tfrac{1}{4}\, h. \frac{3\,\mathrm{R}r + 2\,\mathrm{R}r + r^2}{\mathrm{R}^2 + \mathrm{R}r + r^2}$$

The same formula will serve for the frustrum of any regular pyramid, using the sides of the upper and lower surfaces instead of R and r.

In the solid paraboloid:

$$x = \frac{2a}{3}$$

In a frustrum of a paraboloid:

$$x = \tfrac{1}{3}h\,\frac{2R^2 + r^2}{R^2 + r^2}$$

GREGORY, § 124.

THE MECHANIC POWERS.

54. A Machine is an instrument, by means of which we change the direction or the intensity of a force, or both.

55. Complex machines are formed by the combination of certain simpler ones, that are called the mechanic powers.

Of these, writers on mechanics formerly enumerated six, viz. 1. The Lever. 2. The Wheel and axle. 3. The Pulley. 4. The Inclined plane. 5. The Wedge, and 6. The Screw. To these modern authors add the Funicular machine.

The Lever.

56. A Lever is an inflexible bar, whether straight or crooked, resting upon a fixed point, that is called the *Fulcrum*, around which it can move freely.

In the lever, and in all other machines, when two forces are applied, one of which is intended to countervail or overcome the resistance of the other; the first of these is called the power, the second the weight.

57. There are three kinds of lever according to the respective positions of the power, the weight, and the fulcrum. In the first, the fulcrum is between the power and the weight. In the second, the weight is between the power and the fulcrum. And in the third, the power is between the fulcrum and the weight.

58. In a straight lever, if the power and weight act in parallel directions, they are in equilibrio when they are to each other inversely as their distances from the fulcrum. POISSON, *Additions*.

59. In any lever whatever, and in any direction of the two forces, they are in equilibrio, when they are to each other inversely as the perpendiculars let fall from the fulcrum upon their directions.

These properties of the lever may be demonstrated directly, but they may also be deduced from the theory of parallel forces, and extended to any number of forces whatsoever; in this case, one must be supposed to act at the fulcrum, in the manner described, when speaking of bodies resting upon a surface; and unless the fulcrum of the lever be also its centre of gravity, it will be necessary to take its weight into consideration; this may easily be done, by considering it as a force applied to the centre of gravity of the lever, and acting in a vertical direction.

The lever is of very extensive use in practical mechanics, both in its simple state and combined. The most usual instances of its application simply are as follows, viz.

First kind of lever. The quarry-crow, handspikes, scissars, pokers, pincers, snuffers, &c.

Second kind of lever. The oars and rudders of vessels, cutting knives fixed at one end, doors moving on their hinges, &c.

Third kind of lever. Tongs, sheep shears, a ladder raising against a wall, &c. In this kind of lever, the intensity of the force is diminished, but the velocity is increased. We shall hereafter see that it is extensively employed by nature, in the organization of animal bodies.

60. A Balance is also one of the applications of the first kind of lever. In it the arms are of equal lengths; hence, as equal weights suspended from its extremities will be in equilibrio, it is made use of to measure unknown weights by means of some given standard.

It sometimes happens that the arms of a balance may be made of unequal lengths for the purpose of fraud. If this be suspected, it is only necessary to weigh the article, first in one scale, and then in the other; if the counterpoising weights in these different positions be not equal, take their geometric mean, and it will be the true weight. GREGORY'S *Mechanics*, 133.

A balance, when well constructed, must have the following properties. 1. It should rest in a horizontal position, when loaded with equal weights. 2. It should have great sensibility, so that a very small addition in either scale will disturb the equilibrium. 3. It should soon return to rest, after having been put in motion by a change of the weights.

Of all weighing machines, a good balance is that which can be the most depended upon, in cases where great accuracy is desired.

61. The Steelyard or Statera Romana, is another of the applications of the first kind of lever. Its arms are unequal, and the weight of the substance under experiment is ascertained by the distance of a known weight from the fulcrum.

The steelyard cannot be employed where a great degree of accuracy is desired, but is often convenient from its portability. A very compact and useful form, constructed by Mr. Dearborn of Boston, has been introduced into the customs of the United States.

When the weight to be found, as often happens, is much greater than that of the counterpoise, the centres of the steelyard are not loaded with much more than half the weight borne by those of a balance, employed to weigh the same article; hence, the steelyard may at times be used to weigh very heavy bodies. One has been constructed for this purpose, at the West Point Foundery, which is capable of supporting seven tons, and is sensible of an increase or diminution of 5 pounds.

Other weighing machines have been contrived, to act by a combination of levers. Of these the most elegant and practically useful, is that employed in England for weighing loaded wagons and living cattle; it is called the Platform Balance, and is described by Gregory, in the 2d volume of his mechanics. Likewise by LE SAGE in his *Reccuil des Memoires*, Vol. 2, p. 111.

This has also been constructed with great neatness, but on a smaller scale, at the West Point Foundery.

62. When a beam carrying a weight is supported in a horizontal position by two props, the weights which the props sustain are inversely proportioned to their distances from the centre of gravity of the weight. PLAYFAIR, § 128.

63. A compound lever is a combination of two or more simple levers, the first of which is employed to turn the second; the second the third, and so on. In a compound lever, equilibrium exists when the weight is to the power as the product of all the arms on the same side of the fulcrum with the power, is to the product of all the arms on the same side of the fulcrum with the weight.

The Wheel and Axle.

64. The wheel and axle is a machine that consists of a cylinder and a wheel having a common axis, at the two extremities of which are pivots on which the whole may turn. The power is applied at the circumference of the wheel; the weight to the circumference of the axle.

65. The wheel and axle is in equilibrio when the power is to the weight as the radius of the axle to the radius of the wheel.

The wheel and axle may be considered as a lever, so contrived as to have a continual motion round its fulcrum.

It is not necessary that the circumference of the wheel be continuous, the insertion of spokes into the axle is sufficient; windlasses and capstans are, in consequence, modifications of the wheel and axle.

A winch is also a species of the wheel and axle.

The combinations of wheels and pinions, so useful in mechanics, are reducible to the wheel and axle; their effect may be calculated by the following proposition :

66. In a series of wheels and pinions, equilibrium will exist when the power is to the weight as the continual product of the radii of all the pinions is to the continual product of the radii of all the wheels.

Wheels and axles are also sometimes combined together, by cords passing over one wheel and the following axle. The equilibrium is calculated on the same principle as that of the above proposition.

The number of teeth on the circumference of wheels being to each other as the diameters, the proposition is often expressed thus: there is equilibrium in a system of wheels and pinions, when the power is to the weight as the continual product of the teeth in the pinions is to the continual product of the teeth in the wheels.

67. In forming the teeth of wheels it is of great importance to determine their proper curvature, for the motion ought to be communicated equally, and with as little friction as possible. The most advantageous curve for this purpose seems to be the Epicycloid. The Involute of a circle has also been proposed.

See Camus on *the Teeth of Wheels*—Hachette, *Cours Elementaire des Machines*—Buchanan on *the teeth of wheels.*—In order that the same teeth of a wheel and pinion may coincide as seldom as possible, the numbers of them upon each circumference should be prime to each other. The ordinary method of effecting this is to add one to the number of teeth on the wheel.

The Pulley.

68. The pulley is a wheel moveable upon its axis, and having a groove cut upon its circumference through which a cord passes. It is enclosed in a box or case, that is called its block.

A pulley is either fixed or moveable. In the fixed pulley the block remains stationary, in the moveable pulley it rises and falls with the weight.

A fixed pulley does not alter the intensity of the power, but merely changes its direction.

In a moveable pulley, one end of the cord is fastened to a fixed point; the power is applied to the other, either directly or through the intervention of a fixed pulley; the weight is suspended from the block.

This kind of pulley is reducible to a lever of the second kind, and the power is evidently enabled to balance a double weight.

Fixed and moveable pulleys may be combined together in such a way as greatly to increase the intensity of the power. Each different combination will have a different degree of advantage.

To be more conveniently portable, it is usual to place all the moveable pulleys in one block, and sometimes on one axis; one cord only is employed, which is wound round the moveable pulleys, and an equal number of similarly arranged fixed pulleys, alternately. In this combination, equilibrium takes place when the power is to the weight as one to twice the number of moveable pulleys.

The simplest combination of pulleys is that where a number of moveable pulleys hang in a diagonal line above one another, each doubling the power of the one beneath it. In this, while the number of pulleys increases in arithmetical progression, the intensity of the power increases in geometrical progression. The pulley is most extensively used on ship board.

The Inclined Plane.

69. The inclined plane is a mechanic power, consisting, as its name imports, of a plane making any angle, less than a right angle, with the horizontal plane.

70. The general condition of equilibrium in the inclined plane is, that the power should be to the weight as the height of the plane to the product of its length into the sine of the angle which the direction of the power makes with a perpendicular to the plane. POISSON, § 97, 98.

When the direction of the power is parallel to the plane, equilibrium exists when the power is to the weight as the height of the plane to its length, or as the sine of the plane's inclination to the radius.

If the direction of the power is parallel to the base, equilibrium exists when the power is to the weight as the height of the plane to the base. POISSON, § *Additions.*

The Wedge.

71. The ordinary form of a wedge is a triangular prism of wood or metal. Its section is usually an isosceles triangle with a very acute angle at the vertex, but it may be of any triangular figure whatsoever.

72. In an isosceles wedge, equilibrium takes place when the power is to the weight as the thickness of the wedge to its length.

This may be proved by considering the wedge as composed of two inclined planes, joined together at the base.

73. If three forces be applied perpendicularly to the three sides of a scalene wedge, they will be in equilibrio if their directions intersect each other in the same point, and if they are to each other directly as the lengths of the sides on which they act. POISSON, § *Additions.*

This may be demonstrated independently of the consideration of the inclined plane, and the properties of that mechanic power might be deduced as one of the cases.

The mere theory of the equilibrium of the wedge is of little use in practical mechanics. The wedge is generally urged by percussion, and its effects are then often greater than could be anticipated from the theory.

The only instance when a wedge acts in equilibrio, is in the construction of arches.

The wedge is much used in operative mechanics; it is employed to raise heavy weights to a small height; in all cutting instruments, as knives, chissels; in the teeth of animals, &c.

74. A wedge may have the form of a pyramid as well as of a prism; if the faces are inclined at equal angles to the base, the power and weight are in equilibrio when the first is to the last as the cosine of the angle of inclination to the radius.

Piercing instruments of all kinds are applications of this form of the wedge.

The Screw.

75. The screw is a spiral groove or thread winding round the surface of a cylinder, and cutting all lines drawn upon its surface and parallel to its axis at equal angles.

This spiral may be either on the inside of a hollow cylinder or on the outside of a solid one, and the screw is called in consequence the interior or exterior screw, as the case may be. Both must be combined in the practical application, so that the groove of the exterior receives the thread of the interior screw.

The simple screw might properly be referred to the principle of an inclined plane, for it is no more than an inclined plane wrapped round a cylinder; but in using the screw, it is usual to apply the power to the extremity of a lever fixed perpendicularly to the axis of the screw, and it thus becomes a machine compounded of the lever and inclined plane.

76. In the screw, the power and the weight are in equilibrio, when the first is to the last as the distance between any two threads of the screw is to the circumference described by the point to which the power is applied.

The screw is used for compressing bodies; it thus forms part of the ordinary printing, and of many other presses: It is applied to raise great weights to a small height.

Under the head of hydraulic engines we shall treat of the screw of Archimedes, used for raising water.

In screws constructed of hard materials, the distance between the threads is exactly equal; and as the revolution of the head of the screw corresponds to the passage of it longitudinally through the distance between two of the threads, it may be used for dividing space into very minute parts. On this principle it is used in dividing astronomical instruments, and in *reading off* their divisions. BREWSTER'S *Edinburgh Cyclopædia, article* GRADUATION.

The Funicular Machine.

77. If a cord be attached by one end to a weight, and a force be applied at the other in the direction of its length, no mecha-

nical advantage will be gained. But if the same cord be attached by one extremity to a weight, and by the other to an immoveable point, while a power is made to act at some intermediate point of the cord in a given direction, it will exert a force upon the weight of a different intensity from its own, and draw it in a direction also different. In this case, a rope becomes a machine. Poisson, § *Additions.*

78. If a body be acted upon by forces through the intervention of ropes, the assemblage of ropes is called a Funicular Machine.

If a body be suspended by ropes from fixed points, it is still a funicular machine, for we may suppose powers equivalent in magnitude and direction to be substituted for the resistance of the points. Prony, *Architecture Hydraulique*, § 248.

79. If a funicular machine, composed of ropes attached to each other, perfectly flexible and inextensible, be in equilibrio under the action of any number of forces whatsoever; and if these forces be transported so as to act parallel to their original directions upon a single point in the system, it will be kept in equilibrio under their joint action. Any one of the forces will also be equal and directly opposed to the resultant of all the rest. Prony, *Architecture Hydraulique*, § 247.

80. The curve which a chain or flexible cord forms, when suspended from its extremities and acted upon by gravity, either alone or jointly with other forces, is called the Catenaria.

The properties of the Catenaria may be deduced from those of the funicular machine. See Poisson, § 134 to 151.

PRINCIPLE OF VIRTUAL VELOCITIES.

81. If we suppose the equilibrium of a lever to be for an instant disturbed, and motion to take place, the product of the power into the velocity with which its point of application would then move, is equal to the product of the weight into the velocity of its point of application.

As this motion does not actually take place, the velocity is styled *Virtual.*

This same proposition may be extended to the cases of all the other mechanic powers, and may be generalized as follows, viz.

82. When the forces, however numerous, that act upon any machine are in equilibrio, if each force be multiplied into the virtual velocity of its point of application, estimated according to its direction, the sum of all the products will be equal to nothing.

This is not only true in machines, but in every possible system of forces in equilibrio; it may be expressed by the following equation.

$$Pp + P'p' + P''p'' + \&c. = 0.$$

P, P', P'', &c. being the several forces, and p, p', p'' &c. their respective virtual velocities. POISSON.

This proposition may not only be deduced from the consideration of the equilibrium of machines, but may be demonstrated directly, as is done by LA GRANGE in his Mechanique Analytique. In his method of proceeding, this principle becomes the fundamental theorem of mechanics. See LA GRANGE, *Mechanique Analytique.*

OF FRICTION.

83. Whenever the forces, applied to the several points of a solid body, fulfil the conditions of equilibrium that have been laid down, the body will remain at rest. The weight of the body itself must of course have been included among the forces, as if it were applied to the centre of gravity in a direction perpendicular to the horizon.

As far as our investigations have hitherto extended, it would appear that nothing more is necessary to put the body in motion than the slightest change in the direction or intensity of the forces, producing a difference in the result of the equations of equilibrium. In practice, however, it will be found, that if a body rest on a plane surface, or touch a fixed obstacle, a physical circumstance that has not yet been taken into view will prevent motion following an apparent destruction of equilibrium. This is the *friction* which takes place between the body and the obstacle.

Friction may be regarded as a *passive* force, for it cannot of itself begin motion, and never acts but in opposition to motions communicated by other forces. Poisson, § 126.

84. If a body lie upon a horizontal plane, it will remain at rest, and will press the plane with a force equal to its own weight. If the plane be inclined in a small degree, although the body does not begin to move, the pressure on the plane will be diminished, and a part of the weight of the body will be supported by the friction.

The exact amount of the pressure and the weight supported by the friction may be found by the theorems of the resolution of forces; if W be the weight and i the angle of plane's inclination, P the pressure and w the part supported by friction,

$P = W. \cos. i.$ $w = W. \sin. i.$ Poisson, § 127.

85. If the inclination of the plane be gradually increased until the body is just about to move, the value of w, at that instant, will be the measure of the friction.

If f be ratio of the friction to the pressure,

$$f = \text{tang.}\ i. \qquad \text{Poisson, § 127.}$$

This furnishes a practical method of ascertaining the quantity of friction. It may also be ascertained by finding the weight which by its descent over a pulley is sufficient to draw the body under experiment along a horizontal plane. The latter appears to be the most accurate method, for it has been found that the force necessary to overcome the friction of a body at rest, and set it in motion, is greater than that which is needed to overcome the friction of a moving body.

The most accurate experiments that have been made on the subject of friction are those of Coulomb and Vince: the former gained the prize of the French academy of sciences in the year 1781. The latter published his researches in the Transactions of the British Royal Society in 1785. See THEORIE DES MACHINES SIMPLES par COULOMB, Paris, 1809, and PHILOSOPHICAL TRANSACTIONS ABRIDGED, vol. 15, p. 654.

A summary of the deductions from Coulomb's experiments is to be found in PRONY, § 1173.

86. The retardation produced by friction is an uniform force. VINCE, § 3.

This also agrees with Coulomb's experiments.

87. Friction is greatest between bodies whose surfaces are rough, and is lessened by polishing them. POISSON, § 128.

88. Friction, *cæteris paribus*, is greater between bodies composed of the same material than between bodies composed of different materials. POISSON, § 128.

89. If the rubbing surfaces remain the same, the friction increases with the pressure. Poisson, § 128.

This deduction appears to agree with the experiments of Coulomb, but is contradicted by those of Vince, who found that friction increases in a less ratio than the pressure. Bossut also mentions the same fact, and adduces the case of a ship when launched in support of his opinion.

90. If the weight continue the same, the quantity of surface has no influence on the friction. Poisson, § 128.

This is also denied by Mr. Vince, who found the friction affected, although in a very small degree, by the magnitude of the surfaces in contact. In practice, however, the increase of the friction with the pressure, and its not being affected by the surface, may be assumed as true, without any risk of error.

91. Friction is very much lessened by making the moving body roll instead of sliding. The cause of friction appears to lie in the porosity of bodies; their surfaces are covered with projections and cavities; when one body lies upon another, the excrescences of the one fall into the pores of the other, and it requires a certain force to disengage them. This must be done either by raising the one body from the other, or by breaking down the projections that interfere with the motion. Coulomb, § 96 to 101.

This hypothesis is sufficient to explain all the facts.

The effect produced by polishing is due to the smoothing of the asperities. Oil, tallow, plumbago, &c. fill up the cavities. In rolling bodies, the asperities do not need to be broken down; nor is it necessary to raise the moving body from the other in order to disengage their parts; this is done by the very action of rolling.

Friction is very much increased when a pressure is applied to the moving body sufficient to prevent the prominent parts of it from being separated from, and raised out of the cavities of the other, Upon this principle we may account for the very powerful effect of fixed rollers in shaping and flattening bars and plates of metal.

92. These experiments and laws furnish the principles of the practical methods of diminishing friction.

Friction may, in practice, be lessened in various ways.

(1.) The rubbing body may be drawn in a direction inclining upwards from the plane on which it rests. In this manner a part of the force will be applied to raise the projections of the one surface from the cavities of the other.

(2.) The asperities may be smoothed by the mechanical action of polishing.

(3.) The cavities may be filled up with oil, grease, plumbago, &c.

(4.) The body, instead of sliding, may be made to roll, or if this be impossible, it may be laid upon rollers, or on a wheel carriage. The value of wheels and rollers arises partly from the physical circumstance already mentioned, and partly from a mechanical advantage possessed by the power applied to overcome the friction.

Rollers put under a heavy body diminish the friction in the greatest degree possible if they are spheres or cylinders, and not constrained to move round a fixed axis. In wheels, the conversion of sliding into rolling is less complete. Playfair, § 165.

(5.) Under certain circumstances, friction wheels and rollers may be introduced with great advantage.

Two of the most elegant applications of friction wheels are to be found in the machine of Atwood, and the patent blocks of the late Mr. Garnett, of New-Brunswick, N. J.

(6.) Friction may be lessened by causing heterogeneous bodies to move against one another, instead of homogeneous ones. Besides the above methods, if Mr. Vince's experiments be correct, there are some few instances where advantage is to be derived from a diminution of the surface.

93. Friction is one of the causes which operate to bring bodies moving upon the surface of the earth to rest. A simple knowledge of its action, as a retarding force, is sufficient to show the futility of the hopes of those who seek for the perpetual motion.

94. Although friction destroys motion, and generates none, it is of essential use in mechanics. It is the cause of the stability of machines, and is necessary to the exertion of the force of animals. PLAYFAIR, § 172.

Nails, screws, or bolts, would be of no use in holding the parts of a machine or building together, were it not for their friction. The wedge would be forced back at each interval between the blows that urge it. The pulley would not be moved by the cord passing over it. Arches erected upon the best principles would, if friction did not exist, be forced from their equilibrium, and thrown down by a very trifling weight.

95. When a machine is in equilibrio, under the action of the power and weight, it will not be ready to move until an addition be made to the power that is equal to the friction.

In the lever, if the ratio of the pressure to the friction be as n to 1, if P be the power, W the weight, r the diameter of a cylindrical axis used as a fulcrum, and a the length of the arm to which the power is applied; the expression for the addition required to overcome the friction will be

$$\frac{r}{na}(P + W)$$

PLAYFAIR, § 169.

Of the elementary machines, the lever and inclined plane have the least friction, the screw and the wedge the most.

OF THE STIFFNESS OF ROPES.

96. The resistance which ropes oppose to the effort of a force tending to bend them has a great analogy to friction. It is a constant quantity, being always the same in the same rope under similar circumstances. Prony, § 1206.

The best experiments on the stiffness of ropes were made by Coulomb, and are detailed in the work already quoted; an abstract of them is given by Prony.

97. In ropes the resistance to a power applied to bend them increases with some determinate power of the diameter, which we shall call n.

In new ropes $n = 2$.
In old ropes $n = \frac{3}{2}$. Prony, § 1208.

98. The resistances also are directly as the tensions and inversely as the diameters of the cylinders round which they are wound. Prony, § 1208.

99. The friction of a rope which is wound round a cylinder increases in geometrical progression while the number of turns increases in arithmetical progression.

By means of the above property we may explain the great value of this kind of resistance, when it is wished to oppose a sudden check to the motion of a body, or to make fast the end of a rope that is violently strained. Playfair, § 171.

STRENGTH OF MATERIALS.

No real improvement has been made in the theoretic consideration of the subject of the strength with which materials resist a strain applied to break them, since the time of Gallileo. In [illegible] ders

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In square beams of the same length, the lateral strengths are as the cubes of the sides.

In cylindric beams of equal length, the lateral strengths are as the cubes of the diameters.

If several beams be laid one upon the other, they are not able to bear a greater weight than when laid side by side.

The same rectangular beam is strongest when laid with its narrowest side up. And it is as much stronger in this position than in the other, as the greatest breadth of its section is greater than the least. EMERSON'S *Mechanics*, § 5.

112. If a beam be supported at both ends, and a given weight be laid upon it, the strain upon it will be proportioned to its length.

The strengths, therefore, of beams of the same material, and pressed by equal weights, are inversely as their lengths. But where the power which tends to break a beam is its own weight, its action increases with the square of the length of the beam, for the weight of beams of equal sections is as the length. EMERSON, § 5.

113. A beam whose section is a triangle, and which is supported at both ends, is twice as strong when the edge is uppermost as it is when the opposite side is highest. GREGORY, § 177.

114. The lateral strengths of two cylinders of the same material, and of equal length and weight, one of which is hollow and the other solid, are to each other as the diameters of their ends.

Nature appears to make great use of this principle in her works, where in innumerable instances the strength of substances is greatly augmented without increasing their weight: for instance, in the bones of animals; the feathers of birds; the stalks of reeds, grasses, &c.

In the arts there are very many cases where hollow cylinders may, with great advantage, be substituted for solid ones of equal or even greater weights. This is more especially the case when metallic shafts are employed, for some experiments would appear to show that a hollow metallic beam is stronger than a solid one of the same material and equal diameter. GREGORY, § 172.

106. Upon the same hypothesis from which the above theorems have been deduced, it may be demonstrated that the strength of a beam, in an inclined position, is to its strength in a horizontal position, to resist a vertical pressure, as the square of radius to the square of the cosine of elevation. GREGORY, § 185.

This does not agree with experiment, for, if it were true, the strength of an upright pillar would be infinite. The strength of beams of wood is, however, very much increased, by making the pressure act in the direction of their length.

107. Large beams and bars are in much greater danger of breaking than small ones; and what appears firm and strong in a model, may, when executed on a large scale, fall to pieces by its own weight. Thus we find limits to the extent of the works of art, and even of those of nature. For in all cases whatever, the strength of bodies increases only as the cube of their similar dimensions, while the energy of the power applied to break them increases with the fourth power.

The best experiments on the strength of wood of different sorts are those of Buffon.

Experiments on the strength of wood and iron have been made with much accuracy by Banks in England. They are to be found in Gregory's Mechanics, vol. i. § 191.

Experiments on the strength of white marble, made by Professor Robinson of Edinburgh, are also to be found in the same work.

108. In combining together pieces of wood, or any other materials in machines, or in the erection of buildings; the parts of each piece, and the several pieces, must be so adjusted that the strength may be always proportioned to the strain they are to endure. EMERSON, § 5.

The most important application of this subject is to the theory of architecture. The architecture that is now practised in different parts of the world, or of which remains are to be still seen, appears to have taken its rise from several different sources: Chinese architecture seems to be derived from the tent of their Tartar ancestors; Egyptian architecture, from the caves occupied as dwellings by the first inhabitants of that country; Grecian architecture, from the log hut.

The Pyramid is an improvement upon the earthen tumulus, and the rich ornaments of the Gothic cathedral are copied, in a more costly and lasting material, from the temples of wicker work erected by the Northern nations.

109. When a beam is placed in a perpendicular position, it becomes a pillar, which, if of a circular section and regular proportion, is called a column. By the investigations of La Grange, it would appear that a cylindrical form is the strongest to resist flexure. When, however, we take into view that a column has not only the superincumbent, but its own weight to support, and that the pressure on its lower parts will in consequence be greater than that upon the upper, it is evident that the thickness of the lower parts should be somewhat increased, and this is always done in practice. YOUNG's *Lectures on Nat. Phil.* § 14.

The best outline for a column is a curve that is convex externally. And this is not only the strongest, but the most graceful shape.

When a column is fixed upon a solid base, and without having any weight to support, is exposed to the action of a fluid; the best form for the outline is a curve concave towards the exterior. We find instances of such columns in nature, in the trunks of trees, and at the insertion of their branches. Smeaton, who observed this, has used it with great success in the construction of the Eddystone lighthouse. SMEATON, *History of the Eddystone.*

A wall should also be thinner at top than at bottom; and this is the more necessary, as in most cases the weight is distributed among several floors instead of resting upon the top of the wall. YOUNG, § *Lecture* 14.

OF ARCHES.

The ordinary mode of treating the theory of arches is to consider them as composed of a number of truncated wedges, kept in place by their mutual pressure, and free to move upon each other by the smallest change in the conditions of equilibrium. Friction, as well as the cohesive power of the cement, are left out of the question. The following are some of the deductions from this hypothesis.

110. In any arch composed of a number of wedges, as described above, the wedges will be in equilibrio when they are to each other respectively as the portions cut by the prolongations of their joints from a line passing horizontally through the apex of the arch.

The truncated wedges, of which an arch is composed, are called *Voussoirs.* They are usually of an uneven number.

The central voussoir is called the *Keystone.*

The surfaces where the voussoirs rest upon each other are called *Joints*.

The interior curve of an arch is called the *Intrados*, the exterior the *Extrados*.

Abutments are the masses of masonry that support the arch. The beginning of the arch is called the Spring of the arch, the middle the Crown, and the part between the crown and the spring is termed the Impost. PLAYFAIR, § 224. HUTTON, *Tracts*, vol. i.

111. In a circular arch, the weights of the voussoirs should be as the difference of the tangents of the arcs reckoned from the crown. PLAYFAIR, § 224. HUTTON, *ubi supra.*

The whole mass of masonry is supposed to be cut by prolongations of the joints, so as to extend the voussoirs to its upper surface.

112. If the weights of the voussoirs be all equal, and their lower surfaces plane, the polygon which fulfils the conditions of equilibrium, is the funicular polygon; or if it become a curve, it is the Catenaria.

The equation of the catenaria is

$$y = \mathrm{A} \times \text{hyp. log.} \frac{\mathrm{A} + x + \sqrt{2ax + x^2}}{\mathrm{A}}.$$

A being a constant quantity that can be determined by experiment. See PLAYFAIR, § 228.

The most remarkable property of a catenaria is, that the chain or cord, by whose suspension it is formed, has its centre of gravity lower than if it were disposed in any other line.

When it is wished to construct an arch of pieces of timber, that shall be in equilibrio, nothing more is necessary than to hang a model composed of a system of smaller but similar pieces connected together by pivots, from two points, whose distance bears the same relation to the span of the arch to be constructed as the lengths of the pieces of the model do to those which are to be used. The polygon formed in this way will be similar to a larger one, which, if inverted, will have the thrusts of all its parts in equilibrio. HUTTON, vol. i. p. 20.

If the voussoirs be not of equal weights, an arch of equilibration may be found by hanging from a number of points in a catenaria, cords or chains that represent in weight and magnitude the corresponding parts of the arch to be constructed. The new curve thus formed is an arch of equilibration. HUTTON, vol. i. p. 21. ENCYCLOPEDIA BRITANNICA, Supplement.

The chain bridges constructed in this country by Mr. James Finley, of which there are beautiful examples upon the Brandywine and Nishamony, assume naturally the above form, and as their vibrations are unimpeded, are always in equilibrio. See PORT FOLIO for 1810.

A bridge precisely similar to those of Mr. Finley, has been lately erected over the Tweed in Great Britain, by Captain Samuel Brown. EDINBURGH JOURNAL for October, 1821.

Except in the two cases of a chain bridge, and a balanced roof or arch of timber, the above theory is of no value in practice.

113. When the voussoirs that compose an arch are laid upon one another in the process of building, so far from being free to move upon each other, it is found that they will not slide downwards until the inclination of the joint becomes nearly 40 degrees. GAUTHEY, *Construction des Ponts.*

This is still further increased by the action of the cement, and thus it will happen, that an arch which fulfils none of the conditions of equilibrium of the hypothesis shall be stable, while an arch of equilibration may be crushed by its parts acting in a manner so different from the hypothesis.

114. When an arch of any description is loaded with a weight that is nearly sufficient to break it, experience shows that each half has a tendency to divide into two parts; the joints of the voussoirs near the key-stone opening at the arch, while the joints of those nearest the spring open at the opposite side. LESAGE, *Receuil des Memoires*, vol. 2.

The points of division between the two parts are called the points of rupture. A knowledge of their situation is indispensable in ascertaining the proper weight of piers and abutments to support the pressure of arches. Formulæ for finding them are given by GAUTHEY, vol. 1. p. 325. et seq.

115. The thickness of the key-stone of an arch might be no more than is sufficient to prevent it from being crushed by the superincumbent weight; its breadth no more than will include a mass of material sufficient to resist the horizontal pressure of the two halves of the arch.

In practice, a greater thickness should be given in order to sustain sudden shocks; in almost all bridges, however, the thickness of the keystone is made greater than is necessary.

Perronet's rule for the thickness of the arch at the keystone is as follows, viz:

To one twenty-fourth part of the diameter of the arch, add one foot, and from the sum subtract one line for every foot of span; the remainder will be the thickness of the keystone.

This rule of Perronet's, although it gives rise to a construction of arches apparently more bold than of almost any artist who preceded him, does not give as low a value as might be deduced upon the principle we have laid down.

A bridge constructed by Inigo Jones over the Conway, at Llanrost, in North Wales, has the keystone of its centre arch thinner than any of Perronet's, and it has stood for two centuries.

It has been usual to give piers the same thickness as abutments, and where arches are constructed separately this is necessary, but when two arches press horizontally against each other, the pier may be made much smaller, and yet be sufficiently strong. Narrow piers, in bridges, are attended with the advantage of leaving a wider passage for the water.

116. The principal application of the theory of arches is to the construction of bridges. Five different kinds of material are employed in building them, viz. stone, wood, cast iron, wrought iron in the shape of chains, and ropes.

The magnitude of the arches of bridges must be suited to the locality where they are erected, and will be limited by the strength of the material. The largest arches of stone ever constructed were those of the Bridge of Trajan over the Danube; their span was 180 English feet; this bridge is now in ruins. The span of the Waterloo bridge is no more than 120 feet; that of Neuilly over the Seine, 125 feet.

In wood, arches may be of greater magnitude; the bridge erected by Wernwag over the Schuylkill has a span of 340 feet. The length of the famous bridge of Schaffhausen was 365 feet, but it was divided into two arches.

117. The principles of Dome-vaulting have been investigated upon the same hypothesis as those of arches. The results, however, are of little value in practice. It is sufficient to know that the horizontal pressure of the vault must either be counterbalanced by heavy loads of material resting perpendicularly on its spring, or counteracted by some mechanical contrivance. Thus in the dome of the Pantheon, at Rome, the vault is much thicker near its circumference than at its centre, and its exterior figure a less arc of a greater circle than the interior section, which is a semicircle; in gothic vaults the buttresses are loaded with heavy pinnacles, and the domes of St.

Peter's, at Rome, and St. Paul's, in London, are embraced by strong chains.

The construction of domes is easier and less liable to accident than that of arches. In many cases they may be built without centres of carpentry.

Each course keys itself; they may therefore have large openings in the centre for the admission of light.

MECHANICS.

SECTION SECOND.—DYNAMICS.

OF MOTION.

We have seen that the ideas of Space and Time, are necessarily involved in the definition of Motion.

118. Velocity is the relation between the spaces described and the times elapsed since the motion began. Poisson, § 179.

119. The simplest species of motion is where the direction is rectilineal, and the spaces described in equal times are themselves equal; this is called *Uniform Motion*, or motion with an *Uniform Velocity*. Poisson, 179.

Uniform motions differ from one another in their velocities.

In measuring velocities, it is convenient to take some known portion of time for the unit, in terms of which all other portions of time are to be designated. The unit in general use is the second of time.

120. A body if once set in motion will continue to move forward in a straight line with uniform velocity, until it is acted upon by some new impulse.

Although all bodies near the Earth's surface tend to come at last to rest, however violent their original motion may have been, yet, this is no argument against the above proposition, but rather tends to confirm it; for we find the motion of all such bodies to be opposed by retarding forces; among these may be mentioned the resistance of fluid media, and friction; and we also find that by lessening the intensity of the retarding forces, the duration of the motion may be prolonged to such an extent as to show clearly, that if it were possible to remove them altogether, the motion must continue forever. The truth of this proposition is more evident in the motions of the heavenly bodies, that have for ages been known and observed, and yet, in which no diminution is perceptible. Poisson, § 182.

121. If a body, after having described a given space in a certain time shall subsequently describe a greater space in an equal time, we infer the action of some external cause, or new impulse.

A motion of this sort is called *Accelerated* ; when the velocity increases by equal increments, in equal times, the motion is said to be *Uniformly* or *Equably Accelerated*.

122. So, also, when a body, after having described a given space in a certain time, describes a less space in an equal time, we again infer the action of new impulses, but in a direction contrary to that of the original force.

A motion, where the velocity in equal times decreases, is said to be *Retarded*.

If the decrements in equal times be equal, it is *Uniformly* or *Equably Retarded*.

123. We measure a force by the quantity of motion it is capable of producing, and this in aggregates of matter will depend upon the velocity of the particles and their number.

The term *Mass* is used in Mechanics to signify the quantity of matter contained in a given body. Poisson, § 310.

124. In homogeneous bodies the mass is proportioned to the bulk ; but bodies of different species often contain, under equal bulks, very different quantities of matter.

Density is the comparison between the quantities of matter of different bodies contained under equal bulks.

The difference in density, which is found in different bodies, appears to depend in a great degree upon the interstices that are known to exist between the constituent particles. These interstices are called *Pores*; their number and extent are so great as to render it probable that even in the densest body, the quantity of matter is small when compared with the quantity of empty space. NEWTON's *Optics*, Book 2d.

In applying the principles of Statics to the Dynamics of aggregates of matter, or systems of bodies, the following law, known by the name of the *Principle of D'Alembert*, is of great use.

125. If there be a system of bodies mutually acting in any manner upon each other; and if at any given time we compute the motions that these bodies would have in the succeeding instant, were their mutual action to cease; and if we also compute the motions that, in consequence of their mutual action, they actually have at that instant, the motions that must be compounded with the first of these in order to produce the second, are such as, if they acted upon the system alone, would produce no motion, or would be in equilibrio with each other. PLAYFAIR, § 117. POISSON, § 332. PRONY, § 382.

OF UNIFORM MOTION.

126. The relations between the velocity of a body moving uniformly, the space it describes, and the time of description, may at once be deduced from the definition of Velocity, for if v be the velocity, s the space, and t the time

$$v = \frac{s}{t} \text{ hence } s = t\,v \text{ and } t = \frac{s}{v}$$

A more comprehensive form may be given to this equation, so as to make it capable of comparing the motions of bodies that do not set off from the same point; thus, if s be the distance of a moving body, at some particular period, from a fixed point in the direction of its motion, b the distance of the body from the same point, at the end of the time t, v the velocity as before ; then $s-b$ will be the space described in the time t. And therefore

$$v=\frac{s-b}{t} \text{ and } s=v\,t+b. \qquad (1)$$

The variable quantities s and t may be either positive or negative, according as they represent the situation of the moving body in relation to the given point, at periods before or after that where its place coincides with the point.

If another body move upon the same straight line with a velocity $=v'$, if its distance from the given point $=s'$, and if it move during the time t to a distance $=b'$; the equation of its motion will be $s'=v'\,t+b'$ (2)

By combining these two equations we may solve every question in relation to the relative motion of the two bodies ; if, for instance, we wish to know the instant they meet, then

$$v\,t+b=v'\,t+b'$$

whence we have

$$t=\frac{b'-b}{v-v'}$$

POISSON, § 180.

127. It has been shown that the resultant of two forces, represented in magnitude and direction by the sides of a parallelogram, is represented by the diagonal; hence it is evident, that if a body be acted upon by two forces that would each impel with it uniform velocity, it will, under their joint action, describe the diagonal of the parallelogram of which the forces are sides.

The same is true if it be acted upon by two variable forces provided the ratio of acceleration be the same in each.

GREGORY, § 217, 218.

128. If a number of bodies be moving in any manner whatever,

and if an equal and parallel force act upon each of their particles of matter, the relative motions of the bodies will not be affected.

On this principle we can account for several phenomena; for instance: If a fleet be manœuvring in a current, the evolutions will be performed, and the relative positions of the ships will be the same, as if in still water;

The motions and operations in a ship sailing smoothly and regularly along are performed in the same manner as if the vessel were at rest;

Relative motions upon the Earth's surface are not altered by the rapid motion both of rotation and translation with which our planet is affected in its diurnal and annual course.

OF COLLISION.

The simplest mode in which a uniform motion can be communicated from one body to another, is by impulse.

Their action upon one another is governed by a principle known as the third law of motion, that may be announced as follows, viz,

129. Action and Reaction are equal to each other, and in contrary directions.

This holds good not only when the bodies come into actual contact, but where they act upon one another at any distance whatever. It is nothing more than a different form of the principle of Inertia already laid down, § 12.

130. All the bodies with which we are acquainted are more or less compressible; and when they have been compressed they have a greater or less tendency to recover their primitive form. This tendency goes by the name of *Elasticity*. POISSON, § 421.

131. A Body is said to be perfectly elastic when it recovers its

original form, as soon as the cause that compressed it ceases to act, with a force equal to the compressing force.

Different bodies have different degrees of elasticity, but it does not depend upon their compressibility: the gases are the most easily compressible of all bodies, and are perfectly elastic; while there are bodies easily compressible that have little elasticity, and others that are very difficult to compress, whose elasticity is notwithstanding very great.

132. Although no body is absolutely devoid of Elasticity, yet in treating of the collision of bodies we investigate the laws of the communication of motion as if the bodies under consideration were either absolutely non-elastic or perfectly elastic.

Collision of Non-Elastic Bodies.

133. If A and B be the masses of two non-elastic bodies that move in the same straight line with the velocities a and b, they will, after collision, go on together; and if v be the common velocity after the stroke

$$v = \frac{Aa + Bb}{A + B}$$

If b is negative, in which case the bodies meet each other

$$v = \frac{Aa - Bb}{A + B}$$

If $A = B$

$$v = \frac{a + b}{2}$$

If one of the bodies B is at rest

$$v = \frac{Aa}{A + B}$$

If B be an immoveable obstacle

$$v = 0$$

Poisson, § 422, 423, 424. Playfair, § 70.

It may be proper here to observe, that no body in nature is absolutely immoveable; those which are so considered have their masses infinitely great when compared with the bodies that act upon them; so that they destroy their motion without acquiring a velocity that can be measured. It is thus, for instance, with bodies falling upon the surface of the earth.

From the above formulæ, others may be readily formed to show the quantity of motion lost or gained by each body after collision.

The following rule is, however, more general and easy to recollect.

134. If two Non-Elastic bodies moving in the same straight line impinge against each other; and if the *sum* of their motions before the stroke be divided into two such parts as are proportional to the quantity of matter in the two bodies, each of these parts will respectively exhibit the motion of the corresponding body after the stroke. See Helsham. *Lecture* 4.

135. If the two bodies before collision move in lines, making a given angle with each other; as there is no change produced in the absolutely quantity of motion by the mutual actions of the bodies upon each other, their common direction and velocity, after the stroke, may be easily found.

If the forces of the two bodies be respectively r and s; resolve either of them into two others, t and v, one of which (say t) is parallel, the other (v) perpendicular to the direction of s; if from the point of concourse two lines be drawn, one in the direction of s, and equal to the sum of s and t, the other perpendicular to s, and equal to v; the diagonal of the rectangle contructed upon these two lines will represent in magnitude and direction the joint motion after the stroke; divide it by the sum of the masses, and it will give the common velocity.

Collision of Elastic Bodies.

144. If A and B be two bodies perfectly Elastic, moving in the same direction with the velocities a and b, the velocity of A after impact is

$$\frac{(A - B) \times a + 2Bb}{A + B}$$

and that of B

$$\frac{(B - A) \times b + 2Aa}{A + B}$$

Several consequences follow immediately from these formulæ.

(1) When $A = B$, if the velocity of A, after the stroke, be called v, and that of B after the stroke v',

$$v = b \quad v' = a$$

so that in the case when the masses are equal, the velocities are interchanged.

(2) The relative velocity of the bodies, after the stroke, is equal to their relative velocity before the stroke, but with the contrary sign.

(3) In the shock of perfectly elastic bodies, the product of their masses into the squares of their velocities is the same both before and after the stroke. Poisson, § 428.

145. The product of the mass of a body into the square of its velocity is what has been called the *Vis Viva* of the body.

Some philosophers estimate the force of a body in motion in this way, instead of considering it as due to the mass and velocity simply. The third deduction from the action of elastic bodies upon each other, has been alleged by them in confirmation of their views.

138. In order to find the quantity of motion lost or gained by the collision of two Elastic bodies, we have the following rule: Consider them first as non-elastic, and estimate, by the rule in section 134, the motion that each would have after the stroke Find also the quantity and direction of the motion that would be communicated to each of the bodies, which is of course the difference between its original motion, and that just found. If to the quantity of motion that each body would have after the stroke, if non-elastic, be added the motion communicated to it, upon the same hypothesis, the sum will be the quantity of motion that each body actually has after the impact. HELSHAM, § 5.

139. If an elastic body in motion strike against a greater elastic body at rest, the quiescent body will be carried forward with a quantity of motion greater than that possessed by the striking body; and the striking body will return in a direction contrary to its original one.

It will thus happen, that although there is an increase of the motion, if that communicated to the larger body be alone considered, yet that the *sum* of the two motions, after impact will be exactly equal to what the smaller body had previously.

140. If between two unequal elastic bodies A and C a third, B, be interposed; and if the least A be made to strike on B, with any given velocity, the motion communicated to C. will be the greatest possible when the masses of A, B, and C, are in geometric progression.

If the number of bodies in geometric progression be increased without limit, the quantity of motion communicated to the last may also be increased indefinitely. Notwithstanding this, as all the bodies will move back after collision except the last, the sum of all the motions in the direction of the first motion, is never greater than Aa. The sum of the products of each body into the square of its velocity also remains the same as it was before, namely Aa^2. PLAYFAIR, § 77.

141. If an elastic body strike against an immovable obstacle it will be reflected from it. If the obstacle be a plane, and the body spherical, its direction before and after collision will make equal angles with the plane, but towards opposite sides.

If the bodies be not perfectly elastic, their motions after collision are not such as are determined by § 138. They may however be investigated upon a similar principle.

When the bodies are not perfectly elastic, the sum of the products of the bodies into the squares of their velocities, does not continue the same after collision that it was before; but the quantity of motion estimated in a given direction, remains the same in every case. Playfair, § 80. Poisson, § 525, 428.

142. In investigating the laws of collision, whether of elastic or non-elastic bodies, a property of their centre of gravity is discovered, which we shall hereafter find to be common to the centres of gravity of all systems of bodies that act mutually upon each other; this property is as follows, viz. The State of the centre of Gravity of bodies impinging against each other, either as remaining at rest, or moving uniformly forward in a straight line, is not affected by their mutual action.

OF MOTION EQUALLY ACCELERATED, OR RETARDED.

143. There are in nature two distinct species of forces; those of one description act upon bodies during an inappreciable instant of time, and then abandon them to themselves; they consequently cause a rectilineal and uniform motion. Forces of the other species act without interruption on the moving body during the whole duration of its motion.

If the action be constantly in the same direction, the motion it produces is Rectilineal, but the spaces described in equal times are not equal. Poisson, § 183.

Those motions where the relation between the spaces described, and the corresponding times is constantly changing, are in general terms called *Variable.*

The forces which by their uninterrupted action cause variable motions, whether accelerated or retarded, are in general terms called *Accelerating Forces.*

Although the motion produced by accelerating forces, is neither uniform, nor an assemblage of uniform motions in the same direction, yet, in order to represent a variable motion with more ease, we consider the time as if it were divided into a number of infinitely small parts, and the motion as uniform during each of these small portions ; variable motion is thus resolved into a succession of uniform motions, the duration of each of which is infinitely small, and all the corresponding velocities different. POISSON, § 184.

The changes of motion, however, are always made gradually, and never *per saltum.* See PLAYFAIR, § 95.

By thus decomposing a variable motion into an infinite number of uniform motions, the demonstrations and calculations are simplified, while the evidence of the one, and the accuracy of the other, are not in any way affected. This mode of proceeding is analogous to that made use of in the higher Geometry, where we suppose polygons of an infinite number of sides to be substituted for curved lines.

144. When a body is moving with variable velocity, the motion may at any instant be rendered uniform by removing the action of the accelerating force ; the velocity of the uniform motion thus produced, is called the final velocity acquired by the moving body, or simply its velocity at that instant of time.

145. A variable Motion, where the increments or decrements of velocity are equal in equal times, is said to be uniformly or equably accelerated or retarded. The accelerating force by which it is produced is said to be *Constant.* POISSON § 187.

146. If s be the space described by the action of an accelerating force, if t be time, v the velocity, and g the measure of the accelerating force then,

$$s = \frac{gt^2}{2} = \frac{v^2}{2g} = \frac{t\,v}{2}$$

whence we have

$$t = \frac{v}{g} = \sqrt{\frac{2s}{g}} = \frac{2s}{v}$$

$$v = \sqrt{2gs} = \frac{2s}{t} = g\,t$$

$$g = \frac{v}{t} = \frac{2s}{t^2} = \frac{v^2}{2s}$$

If the time of the motion's continuance be unity,

$$s = \frac{g}{2} \text{ and } v = 2s$$

hence;

147. The velocity acquired by a body acted upon by a constant accelerating force during a given time, is such as would cause it to describe, with uniform motion, twice the space in an equal time.

g The measure of the accelerating force is equal to twice the space described from rest, in the unit of time. Poisson, § 187.

148. The spaces described are as the squares of the times, and as the squares of the final velocities.

If the times be as the series of natural numbers, the velocities acquired will be as the series of even numbers; the whole spaces as the series of square numbers; and the spaces corresponding to each unit of time as the series of odd numbers.

Gregory, § 243.

The motion of heavy bodies in vacuo, affords an instance of motion with an equably accelerated velocity.

Of the Attraction of Gravitation as an Accelerating Force.

149. The action of the attraction of gravitation is not confined to the surface of the Earth, but pervades the whole Universe.

This great law of nature was first discovered by Sir Isaac Newton, and is confirmed by every fact, and by the result of all calculations.

Newton, considering that the power of gravity acts equally upon all matter upon or near the surface of the Earth, that it is not sensibly less on the tops of the highest mountains, that it affects the atmosphere to its utmost limits ; could not believe that its influence ceased abruptly, but was induced to think it was a more general principle, and extended to the heavens ;

By computing the distance which the moon falls from a tangent to her orbit in a minute of time, and comparing it with the distance a heavy body falls near the surface of the earth, in a second, he found that both observed the same law ; whence he inferred that the cause was the same in both. Having thus shown that the attraction of gravitation extended to the moon, it was easy to infer that it included in its influence all the planets of the Solar System.

M'Laurin, *Philosophical Discoveries of Newton.*

150. The Attraction of Gravitation is common to all ponderable bodies, and mutual between them.

It is proportionable to the quantity of matter in the body.

Its intensity decreases as the square of the distance from the centre of attraction increases.

When a heavy body falls near the surface of the earth, it might at first appear that it did not act upon the earth, for the latter remains apparently immoveable ; nor was it for a long time perceived that large masses of matter projecting above the level of the earth, produced any change in the direction of gravity. These appearances led philosophers, before the time of Newton, to suppose this species of attraction to be confined to the earth, and not to be a general property of matter. When, however, a heavy body falls, the law of the equality of action and re-action, is not less true than in any other case of the communication of motion; but the mass of the earth is so enormous, compared with all the bodies that project from, or move near its surface, that the motion communicated to it by their attracting forces is infinitely small.

It has been found by astronomical observations, taken on opposite sides of lofty mountains, that the direction of gravity, as shown by the plumb-line, was altered by their action.

This phenomenon was first remarked by the French Academicians, who visited South America for the purpose of measuring the length of a degree of the meridian.

Dr. Maskelyne, for many years the British Astronomer Royal, made a series of astronomical observations near Schehallion a mountain of Scotland, by which the amount of the deviation of the plumb-line from the true vertical was noted. An exact survey was, at the same time, made of the mountain, from which its cubic dimensions were ascertained. The specific gravity of the substances composing it were also accurately determined. From these data, Dr. Hutton has inferred the mean density of the earth to be about five times as great as that of water.

Dr. Hutton has more recently proposed to make an experiment of the same nature on the opposite sides of one of the pyramids.

The density of the earth once known, Physical Astronomy furnishes us with the means of determining that of the Sun, and all the other planets of the system.

See HUTTON'S *Tracts*. vol. 2.

Bodies falling near the surface of the earth are retarded by the resistance of the air, and thus it will appear that dense bodies fall more rapidly than those which are rarer; if, however, a dense and rare body fall together in the exhausted receiver of an air pump, they will both reach the plate of the pump at the same instant of time, showing that the action of gravity varies with the quantity of matter.

The law of the decrease of gravity with the increase of the squares of the distances, was deduced *a priori*, by Newton; it is found to agree exactly with observation.

151. If Gravity were a constant force, its action upon a body falling freely near the surface of the earth, would produce an equably accelerated motion.

The theorems that have been laid down to express the relations of the several circumstances attending a motion of this sort, will, therefore, be applicable to the motion of a falling body, upon the hypothesis that the force of gravity is the same in every part of the space described by the falling body, and that it is not acted upon by any other force.

The force of gravity decreases in fact, according to the law just laid down; but this decrease is so slow, that there is no error in supposing it a constant force.

The resistance which bodies moving near the earth meet with from the medium in which the motion is performed, is an error of greater magnitude, but the exact law which this retarding force follows, has been ascertained. It is the usual, and best mode of proceeding to consider the motion of falling bodies, first, as taking place in vacuo, and then to apply a correction for the air's resistance.

152. The distance a heavy body falls in the first second of time is $16\frac{1}{12}$ feet, and it acquires a velocity that would carry it uniformly through $32\frac{1}{6}$ feet in the same space of time : *g* the measure of the accelerating force in the formulæ of § 146. is therefore equal to $32\frac{1}{6}$ feet.

If it were 32 feet exactly, which is near enough for all cases where no great accuracy is required, these formulæ would become

$$s = 16t^2 = \frac{v^2}{64} = \tfrac{1}{2}\,t\,v$$

$$t = \frac{v}{32} = \frac{1}{4}\sqrt{s} = \frac{2s}{v}$$

$$v = 8\sqrt{s} = \frac{2s}{t} = 32\,t$$

The value of this force varies in different latitudes, as we shall see hereafter.

By the *velocity due to a given height*, is understood the velocity a heavy body would acquire in falling from that height, and by the *height due to a given velocity*, is understood the height from which a body must fall to acquire the velocity.

153. In order that a body projected directly upwards, shall rise to a given height, it must receive an initial velocity equal to that due to the given height.

The velocity will be uniformly retarded.

The times of ascent and descent will be exactly equal.

The formulæ § 146. and § 152. apply equally well to bodies projected upwards, and to falling bodies; only the times must, in the former case, be reckoned from the end of the motion instead of the beginning.

154. If two bodies, A and B, (of which A is the heavier,) be suspended over a pulley, moveable without friction about a fixed

centre, A will descend, and be accelerated by a force proportioned to

$$\frac{A - B}{A + B}$$

or to the difference of the weights divided by their sum.
Playfair, § 88. Poisson, § 335.

If g be the force of Gravity, the formula that will express the velocity of A, at any given time after it begins to descend will be

$$v = \frac{A - B}{A + B} gt$$

It is upon this principle that Atwood's machine for investigating the laws of motion of falling bodies is constructed; although the friction cannot be entirely removed, it is rendered so very small by making the axis of the pulley rest upon friction wheels, that the action of this retarding force may be omitted in the investigation without sensible error.

The resistance of the air may likewise be disregarded, for the experiments are made at such small velocities, that the motion is but little affected by this cause of disturbance.

There are two other ways of ascertaining the value of g: one is by direct experiments upon bodies falling from a great height; in this way the resistance of the air is of a great magnitude; the other is by means of the pendulum, the rationale of which shall be explained hereafter.

155. If an arm of an inflexible substance be suspended from its centre of gravity in a horizontal position, by means of a very slender thread, or metallic wire, it will remain in one position, determined by the action of the thread by which it is supported. But any force of sufficient intensity to twist

the thread will be powerful enough to change the direction of the horizontal arm.

There is an instrument invented by Coulomb that acts upon this principle ; it is called The Balance of Torsion, and has, among other uses, been applied to determine the ratio between the attracting power of the Earth, and that of bodies of known density near its surface. Cavendish, an English Philosopher of eminence, conducted a very accurate set of experiments with this view ; from his investigations, it would appear that the mean density of the earth was 5½ times as great as that of water. PHILOSOPHICAL TRANSACTIONS, ABRIDGED, vol. XVIII.

From this, as well as the Observations at Scheballion, it is known that the mean density of the earth is greater than that of any portion of its surface, and that it probably increases in density towards the centre, thus refuting the idea of those persons who have ignorantly imagined the earth to be hollow.

OF BODIES DESCENDING BY THE ACTION OF GRAVITY DOWN INCLINED PLANES.

156. The force that accelerates the motion of a body down an inclined plane is to the force of gravity as the height of the plane is to its length, or as the sine of the angle of the plane's inclination to radius.

The motion of a body down an inclined plane is uniformly accelerated.

The space through which a body descends from rest upon an inclined plane is to the space through which it would have fallen freely by the action of Gravity, as the height of the plane to its length, or as the sine of the plane's inclination to radius.

The velocities which a body would acquire, in descending down an inclined plane, or in falling through its altitude, are equal ; and, consequently, the velocity acquired by a body in falling from rest through a given height, is the same whether it fall freely, or descend along a plane of any inclination whatsoever.

The time in which a body descends along an inclined plane is to the time in which it would have fallen freely through the perpendicular altitude, as the length of the plane to its height, or as radius to the sine of inclination; and in planes of equal altitudes but of different inclinations the times are as the lengths of the planes.

In planes similarly inclined, the times of descent are as the square roots of their lengths, or of their heights.

If the chords of a circle terminating at either extremity of a vertical diameter be considered as inclined planes they will all be described by a heavy body in the same space of time, and this will be equal to the time of fall through the vertical diameter.

The velocities which bodies acquire in descending along chords of the same circle are as the lengths of the chords.

157. If a body descend along a system of contiguous planes without suffering any loss of motion at the angles, it will acquire a velocity equal to that it would have acquired in falling freely through the height of the plane, and sufficient to carry it up an equal and similar system of planes to an equal altitude, and in a time equal to that of the fall.

It is not, however, true, that a body can descend along a system of inclined planes, without receiving a shock at each of the angles, and it may be demonstrated that the loss of motion at each angle is proportioned to the versed sine of the inclination of the planes. Playfair, § 196.

If the number of planes be indefinitely increased, their inclination will be diminished until the sum of the versed sines shall be less than any given magnitude. The above proposition will therefore be true of bodies descending upon curved surfaces.

158. If two heavy bodies descend, without their motion being

checked at the angles, over two similar but unequal systems of inclined planes, the times of descent will be as the square roots of the lengths of the systems ; and if they descend over two similar curved surfaces, the times will be proportioned to the square roots of the length of the surfaces.

When bodies decend over similar arcs of circles, the times will be as the square roots of the diameters.

In investigating the above results, the friction of the descending bodies upon the inclined planes, and the resistance of the air, are not taken into account.

If a body be suspended from a fixed point and made to vibrate, its motion will resemble that of a body descending over a curved surface. The retardation produced by friction on the plane will be removed, and another of less magnitude, viz, the friction upon the point of suspension will begin to act. A body suspended in this manner is called a Pendulum.

OF PENDULUMS.

The laws of the motion of Pendulums are deduced from the consideration of the vibrations of a gravitating body of indefinitely small size, and suspended by an inextensible thread supposed void of Gravity. This is called a *Simple Pendulum.*

The motion of such a pendulum in one direction from a state of rest until it begins to return in an opposite direction is called an oscillation.

159. If a Pendulum were constantly to perform equal arcs of the same circle in its oscillations they would be all isochronous. When, however, a pendulum is set to vibrate, the arcs it describes become gradually less in consequence of the air's resistance and the friction upon the point of suspension ; nor has any uniform maintaining power been yet applied.

160. In a pendulum vibrating in very small arcs of a circle the accelerating force is nearly proportioned to the distance of the point where the oscillation commences from the lowest point of the arc.

The accelerating force is actually proportioned to the sine of half the arc described in an oscillation; but this ratio finally becomes the same as that of the arcs themselves.

161. The vibrations of the same pendulum in very small circular arcs are nearly isochronous. PLAYFAIR, § 198.

The isochronism of vibrations in small arcs of the same circle has been demonstrated by some upon the principle that the half of the arc finally corresponded with its chord. And as the times of descent along all the chords were equal, it was inferred that the times of vibration in small arcs were equal also. This demonstration is not satisfactory, for it can be shown that the more nearly the chord and arc approach in magnitude, the greater is the difference in the time of describing them.

See § 164.

162. When a pendulum vibrates in very small circular arcs, if T be the time of an oscillation p the ratio of the circumference of a circle to its diameter l the length of the pendulum, and g the measure of the force of gravity, the time of an oscillation may be found nearly by the following formula.

$$T = p\sqrt{\frac{l}{g}}$$

If l and T be given

$$g = \frac{lp^2}{T^2}$$

when $T = 1$

$$g = lp^2$$ POISSON, § 272.

It is by means of the latter formula that the intensity of gravity, or the value of g, may be determined at any point upon the surface of the earth from observations upon the Pendulum.

The times of the vibrations of pendulums of unequal lengths are respectively as the square roots of their lengths.

The time of an oscillation is to the time of the fall of a heavy body through half the length of the pendulum as the circumference of a circle is to its diameter.

The lengths of pendulums are inversely as the squares of the number of their respective vibrations in equal times.

The force of gravity in any place whatsoever is proportioned to, and may be measured by, the length of the pendulum vibrating seconds.

163. If a pendulum be placed between two Cycloidal cheeks, and made to vibrate, it will describe in its oscillations arcs of a cycloid identical with that which would be formed by an union of the two cheeks.

164. A Pendulum being thus made to vibrate in a cycloid is at every moment accelerated by a force proportioned to the arc of the cycloid intercepted between it and the lowest point of the arc.

Hence the vibrations of a pendulum in a cycloidal arc whether great or small are all isochronous. PLAYFAIR, § 208.

165. If l be the length of the pendulum vibrating in a cycloid, the time of a vibration whether the arc be great or small will be

$$p\sqrt{\frac{l}{g}}$$

This is the same expression as that deduced for the vibration of a pendulum in a very small circular arc. When, however, the circular arc becomes greater, the value of the time of an oscillation will no longer be the same.

166. The times of vibration in a cycloidal and in a circular arc, and twice the time of descent in the chord of half the circular arc are as

$$1,\ 1+\frac{a}{8r},\ \text{and}\ \frac{1}{.7854}$$

The first and last are constant quantities, but the middle one varies with the extent of the arc of vibration of which a is the versed sine, while r is the length of the pendulum. Hutton's *Tracts*, vol. 3.

Hence the time in the cycloid is the least, and that in the chord the greatest.

167. There is another singular property of the cycloid, viz, that of being the curve of swiftest descent; for the line in which a heavy body moves by the action of Gravity from one point to another, that is neither in the same vertical nor on the same horizontal plane, in the shortest possible time, is an arc of a cycloid having for its base a horizontal line drawn through one of the given points and intersecting the vertical passing through the other.

Poisson, § 288.

168. The simple pendulum, such as it has been defined above, and from the consideration of whose properties the foregoing laws have been deduced, cannot exist in nature, inasmuch as we cannot abstract from the figure and extent of the body or bodies of which it is composed. The Pendulums on which experiment may actually be made are called compound pendulums.

169. The *Centre of Oscillation* in a compound pendulum is the point in which, if all the matter of the pendulum were supposed to be collected and attached by an inflexible line void of weight to the point of suspension, the vibrations would be isochronous with those of the compound pendulum itself.

The distance, then, between the point of suspension, and centre

of oscillation of a compound pendulum, is equal to the length of an isochronous simple pendulum.

Hence we call the distance between the centre of oscillation and the point of suspension of a compound pendulum, the *Length*.

170. The distance of the centre of oscillation of a pendulum composed of several bodies from its point of suspension may be found by dividing the sum of the products of each body into the square of its distance from its point of suspension. by the sum of the products of each body into its distance from the point of suspension.

If the bodies be A, B, C, &c. the distances of their centres of gravity from the point of suspension a, b, c, &c. and x the distance between the point of suspension and the centre of oscilla-

$$x = \frac{Aa^2 + Bb^2 + Cc^2 + \&c.}{Aa + Bb + Cc + \&c.}$$

x will, of course, be the length of an isochronous simple pendulum. When any of the bodies are situated above the point of suspension their distances from it must be accounted negative quantities. PLAYFAIR, § 201.

171. As we may consider that any compound pendulum whatever is composed of an infinite number of small parts, the above theorem may be applied through the intervention of the fluxional calculus to determine the situation of the centre of oscillation of any regular figure.

In a cylinder suspended from the centre of one of its circular bases, if a be the length of the cylinder, r the radius of the base,

$$x = \frac{2a}{3} + \frac{r}{2a}$$

If the thickness of the cylinder be diminished until it become a rod of insensible diameter, there will be but little error in taking

$$x = \frac{2a}{3}$$

In a cone suspended from its vertex, if a be its altitude, and r the radius of its base,

$$x = \frac{4a}{5} + \frac{r^2}{5a}$$

if $r = a$, which is the case in a right cone, the expression will become $x = a$
hence, if a right cone be suspended by its vertex, the centre of oscillation is in the middle of its base.

If a homogeneous sphere, whose radius is r, be suspended from a point whose distance from its centre is d,

$$x = d + \frac{2r^2}{5d}$$

If the point of suspension be in the surface of the sphere,

$$x = \frac{7r}{5}$$

GREGORY, § 311. PLAYFAIR, § 202, 203, 204.

172. The centres of suspension and oscillation are convertible points; that is to say, if the centre of oscillation of a compound pendulum be made the point of suspension, the former point of suspension will become the centre of oscillation.

This remarkable proposition, which was demonstrated by Huygens along with the other properties of the centre of oscillation, remained without any practical application until the present time, when Kater has applied it with great success to the measurement of the lengths of pendulums.

173. There are three several purposes to which pendulums

have been either actually applied, or proposed to be applied, viz. to the measure of time, to the determination of the force of gravity, and as a universal standard of measure.

Galileo was the first who observed the isochronism of the motion of pendulous bodies. Huygens, who investigated the subject more closely, discovered, that as a pendulum was gradually brought to rest by friction round the point of suspension, and by the resistance of the air, the time of its oscillations became less. To obviate this inconvenience he proposed to make the pendulum vibrate in the arc of a cycloid. However good this plan may be in theory it was found impossible to reduce it to practice; but the consideration of motion in a cycloid led to the improvement of making the arcs described by pendulums as small as possible.

As a measure of time, pendulums are attached to instruments known by the name of *Clocks*, that serve the two fold purpose of counting the vibrations of the pendulum, and restoring to it at each oscillation as much force as it loses by the retarding forces that act upon it; this subject shall be treated of in another place.

174. The usual form of a pendulum is a lenticular body attached to an inflexible rod of some metallic substance. As it is found that all metallic substances are affected by alternations of heat and cold, the rate of a clock regulated by a pendulum of this sort must be constantly varying. Means have, however, been found of remedying this defect, the general principle of all of which is, to construct the pendulum of two substances combined in such a way that the expansion of the one upwards shall *compensate* that of the other downwards, and vice versa.

The most remarkable of the contrivances for this purpose are the *Mercurial Pendulum* invented by Graham, and the *Gridiron Pendulum* invented by Harrison. The invention of the last is also claimed by French artists.

In the Mercurial pendulum the rod is a simple metallic verge, the bob is composed of a glass vessel containing mercury. The quantity of the latter is so adjusted that its expansion upwards in the containing vessel shall exactly counterbalance that of the rod downwards; the centre of oscillation is thus constantly maintained at the same distance from the point of suspension. Graham investigated with great care the relative expansions of the solid metals, but found them so nearly similar as to prevent his succeeding in a compensation by making the bob of one metal and the rod of another, and appears to have abandoned the idea of employing them. Harrison, on the other hand, after observing that the relative expansions of brass and steel were nearly as 5 to 3 thought of applying the two metals in the *rod* of the pendulum. The rod of his pendulum, instead of being a single metallic verge, is composed of nine bars, alternately of brass and steel, of such proportionate lengths that by their motion in opposite directions the common length of the rod remains constant.

The form of the Mercurial pendulum has been changed, and the instrument improved of late by that eminent maker of philosophical instruments, Troughton. He has also improved the construction of the Gridiron pendulum, and in one of his methods, where the compensating bars are inclosed in a tube, he has made his clocks more fit to be carried from place to place without risk of the adjustment being altered.

Berthoud and Leroy, in France, have also constructed several species of compensation pendulums, the principle of which does not differ from that of Harrison. See *Rees' Cyclopedia, Article Pendulum.*

175. If a sphere of Platina be suspended by the slenderest wire of iron that is capable of supporting it, we may without any great error abstract from the weight of the latter; if the whole Pendulum be then actually measured, the position of the centre of Gravity may be found by the formula.

$$x = d + \frac{2r^2}{5d}$$

176. If a pendulum of this sort, nearly twice the length of a second's pendulum, be set to vibrate, and the number of its vibrations in a given time ascertained, the true length of the second's pendulum may be thence deduced. To ascertain the number of vibrations in a given time, it is by no means necessary to count them; for if the experimental pendulum be suspended in front of the pendulum of a well-regulated clock, if their planes of vibration be parallel, and their axes both in one plane perpendicular to the horizon and to the plane of vibration, and if the pendulum be nearly, but not exactly some known multiple of the length of a second's pendulum, no more will be necessary than to observe the intervals between the times of coincidence, and hence we may infer the number of vibrations without danger of error.

DELAMBRE *Astronomie*, cap. XXXV. § 125.

When the length of the pendulum and its number of vibrations have been thus determined a correction may be applied for the weight of the suspending wire.

DELAMBRE, *ubi supra.*

It was in this manner that the members of the French Institute determined the length of the pendulum, while measuring an arc of the Meridian. See DELAMBRE, *Base du Systeme Metrique.*

The length of a Pendulum vibrating seconds in the latitude of Paris was found to be 443.296 lines old French measure or 0.993977 metre of the new.

177. If a pendulum be so constructed that it may be suspended at pleasure upon knife-edges either from one extremity, or from a point not far distant from its supposed centre of oscillation; and if a weight slide along the pendulum until its vibrations when suspended by either knife-edge are equal, the position of the second knife-edge will be the centre of oscillation.

The position of the centre of oscillation being thus determined, its distance from the centre of suspension admits of accurate measurement.

This appears to be the best possible method of measuring the length of the pendulum. It was the invention of Captain Kater. His mode of observing the coincidences of the oscillations of the experimental pendulum, and that of the clock although founded on the same principles, appears more likely to be accurate than that of the French Philosophers.

According to his observations, the length of a pendulum vibrating seconds in lat. 51° 31′ 8″, 4 N. when reduced to the level of the sea is 39.1386 English inches.

Philosophical Transactions, 1818. Edinburgh Review, *September*, 1818.

178. From the length of the pendulum as thus found g the measure of the force of Gravity may be at once found by the formula.

$$g = p^2 l$$

If l be taken as $39\frac{1}{6}$ inches, from which it does not sensibly differ

$$g = 386.14 \text{ in.} = 32\tfrac{1}{6} \text{ ft. nearly.}$$

Hence the distance a heavy body falls by the force of gravity in a second of time is $16\frac{1}{12}$ feet.

It is obvious that these numbers are not exactly correct, but approximations only; they are, however, those most commonly used in the English Philosophical Works.

179. When experiments are made upon the length of the second's pendulum with a view of determining the force of gravity in different latitudes, it has been found to vary in such a manner as to show that the Intensity of Gravitation is greater in high than in low latitudes.

The following lengths of pendulums are taken from late observations of Captain Sabine

	Inches.
In Lat. 60° 09′ 42″ N	39.169.
In Lat. 70° 26′ 17″ N	39.198.
In Lat. 74° 47′ 12″ N	39.207.

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The explanation of the cause of this variation in the Gravitating force must be deferred until we treat of *Central Forces.*

180. Since it is possible, without much difficulty, to measure the length of an experimental pendulum, and thus infer the length of one beating seconds, and since this length depends upon the force of Gravity which is invariable at the same place; we might hence deduce an universal standard of linear measure easily accessible, and not liable to change.

Having a standard of linear measure, its square might be taken for the unit of superficial measure, its cube for the unit of measures of capacity; and the weight of its cube filled with pure water as the unit of weight.

181. The properties of a good standard of measure are,

(1.) That it should be easily accessible.

(2.) That it should be invariable.

(3.) That it should be so large that the measures in ordinary use should be deduced from its aliquot parts rather than from itself or from its multiples; for thus any error in the standard would be much lessened.

The pendulum possesses the two first properties in the greatest perfection, but it is far inferior in the last to the unit proposed by the Institute, and actually adopted by the government of France. The unit of the new French measure is a quadrant of the meridian; the standard of length is its ten millioneth part, and is called the metre.

The divisions of the new French measure were all intended to be decimal, but inveterate habits and causes perhaps actually founded in nature have induced the restoration of the old divisions and names in the descending scale, and their application to the new measure.

The measurement of an arc of the Meridian, sufficient to determine its length with accuracy, is an operation that requires much time and labour, and is attended with great expense. For this reason the French Philosophers propose to use the pendulum to recover their unit, in case it should be lost or become changed by use. Thus recurring to the very instrument they had at first rejected, as a standard.

The example of the French teaches us that it would be fruitless to attempt the change of the measures and weights at present in use; all that can reasonably be hoped to be effected is to reduce them to as few denominations as possible, and fix their ratio to some standard in nature, of which the pendulum vibrating in some particular place is the best.

See DELAMBRE *Base du Systeme Metrique.* ADAMS' *Report.* The several REPORTS to the British Parliament. SKENE KEITH'S *pamphlet on weights and measures.* JEFFERSON'S *Report.*

OF THE MOTION OF PROJECTILES.

182. If a heavy body be projected near the earth's surface in a direction not perpendicular to the horizon, it will describe a parabola, whose plane is vertical and whose axis is perpendicular to the horizon.

In the demonstration of this proposition we suppose the directions of the force of gravity at all points within the range of the projectile to be parallel; and we abstract from the resistance of the air.

This proposition was first discovered by Galileo; for his geometrical demonstration see GREGORY, § 248.

For an analytic demonstration, see POISSON, § 229.

If v be the initial velocity, h the height due to that velocity, g the measure of the force of gravity, d the horizontal range, and E the angle of elevation.

$$h = \frac{v^2}{2g} \text{ and } d = \frac{v^2 \text{ sine } 2E}{g}$$

Poisson, § 230.

The horizontal range varies with the sine of twice the angle of elevation, it is of course greatest at 45°; and the ranges with angles equally distant from 45° are equal.

The greatest height to which a projectile rises is as the square of the velocity multiplied into the square of the sine of elevation. The time of flight is as the velocity multiplied into the sine of the angle of elevation. Playfair, § 92.

When the above theory is applied to the motion of military projectiles it is found to differ widely from what actually takes place, for projectiles that according to theory should range in vacuo 20 to 30 miles, do not range in the air more than 2 or 3. This difference between the real and theoretic motion of projectiles arises from the resistance of the air.

183. The resistance of the air to the motion of a projectile increases with the square of the velocity.

With this new element in addition to the two on which the parabolic theory is founded, (viz, the projectile force and that of Gravity,) it has been attempted to investigate the curve that a projectile would actually describe. But the complete integration of its equation has been found beyond the reach of the most subtle calculus. See Poisson, § 234.

The two branches of the curve that would be described by the joint action of the above 3 forces are not similar to each other.

The base of the ascending branch is longer than that of the deoending.

The assymptote of the descending branch is a vertical straight line.

The time in the descending branch is greater than in the ascending. The velocity at the end of the descending branch is less than the initial velocity.

Poisson, § 230. Lallemand's *Artillery.*

184. Beside the resistance of the air in front, the ball is farther retarded by a want of support or *non-pressure* from behind. It is known that air rushes into a vacuum with the velocity of 12 or 1300 feet per second; hence, when the initial velocity of the ball is greater than that quantity, it leaves a vacuum behind it.

The greatest velocity that can be given to a military projectile is about 2000 feet per second; but this is not attended with much advantage, for it is speedily reduced to the above limit.

The effects of these resistances vary in every possible manner in relation to the velocity, the diameter, and the weight of the projectile; hence in order to form rules for practical gunnery it would be necesssary to make experiments upon every species of ordnance, and every size and quality of projectile.

Hutton, *Tract* 37.

The best experiments upon these subjects are those of Hutton, Robins, and Count Rumford.

185. When gunpowder is fired, a permanently elastic fluid is generated, that expands itself both in consequence of its elasticity and of the high temperature at which it is produced. These two causes united appear to give it a force at least 2000 times as great as the pressure of the atmosphere; and it expands itself with a velocity of 5000 feet per second. Hutton, *Tract* 37.

This holds good in the usual mode of firing ordnance, where the ball is free to move the moment the gunpowder begins to inflame. Where the gunpowder is compressed by a heavy weight the experiments of Count Rumford are stated to have shown that it has exerted a force equal to the prodigious amount of 100,000 atmospheres. RUMFORD, *Philosophical Papers.*

This however far exceeds the limits of credibility, but even if the force were much less the effects of small quantities of powder closely confined, as in Military mines, in blasting rocks, &c. might thus be explained. And hence we may see the danger of ill bored cannon and elliptic balls, of obstacles in the barrels of ordnance, and of using wads of an adhesive substance. To the latter cause may perhaps be attributed the bursting of many guns in the naval service of the U. S.

The instrument invented by Mr. Robins to ascertain the velocity with which balls are projected by military ordnance is called the *Ballistic Pendulum.* The same apparatus was also employed by Hutton and Count Rumford. The ball is made to strike a heavy block of wood suspended from a centre; the velocity is thus changed into one that can be easily measured.

186. If v be the velocity with which the ball strikes the pendulum, b the weight of the ball, p the weight of the pendulum, g, the distance from the point of suspension to its centre of gravity, o the distance between the centres of suspension and oscillation, i the distance from the centre of suspension to the point of impact, c the chord of the arc described and r its radius,

$$v = 5{,}6727\, g\, c\, \frac{p+b}{b\, i\, r} \sqrt{o}$$

HUTTON, *Tract* 34.

The position of the centres of gravity and oscillation are determined experimentally See HUTTON, *ubi supra.*

Hutton also measured the velocity of the ball by observing the recoil of the gun, on the following principle.

187. If a ball be discharged from a piece of ordnance, the velocity of the ball will be to that of the gun, as the weight of the gun to the weight of the ball. HUTTON, *Tract* 34.

This is obviously an immediate deduction from the principle of the equality of action and re-action.

188. The motion produced upon a ball by gunpowder inflamed in a piece of ordnance is accelerated as long as the ball remains within the barrel. But the acceleration is not uniform, for the elasticity of the fluid diminishes with the increase of the space it occupies, and with the diminution of its heat; the elastic fluid also propels the ball by following it, and acts with a force proportioned to the difference of their velocities. From these causes, the effect of gunpowder does not increase in as high a ratio as the space it occupies. Hutton's experiments make the velocity to increase with the 5th root of the length of the piece. HUTTON, *Tract* 37.

189. The velocities communicated to balls of equal weights from the same piece and with unequal quantities of powder are as the square roots of the quantities of powder. HUTTON.

190. The velocities communicated to balls of different weights by equal quantities of powder are nearly in the reciprocal ratio of the square roots of the weights of the balls. HUTTON.

191. The firing of the gunpowder, if of good quality and perfectly dry, takes place within a space of time that is very small, but which is not actually instantaneous. HUTTON, *Tract* 37.

This agrees fully with the experiments of Rumford.

192. The deviation of Military projectiles from the path pointed out by the parabolic theory is not confined to the vertical plane, but they are found to be diverted to the right or to the left, and sometimes cross the vertical plane several times in the course of

their flight. One cause of this is the quantity of *windage*, in consequence of which the ball is driven from one side to the other of the bore, and thus acquires a motion around an axis that does not coincide with the axis of the piece; another cause is to be found in the want of sphericity in the balls, and of a true bore in the piece. HUTTON, *Tract* 37.

193. The depth to which a ball penetrates in wood or earth, is as the density and diameter of the ball and the square of the velocity, divided by the strength or resisting force of the obstacle. Hence,

If equal balls be discharged against the same obstacle the depths of the cavities are as the squares of the velocities.

If unequal balls of the same substance be discharged against the same obstacle with equal velocities, the depths of the cavities are as the diameters of the balls.

An 18 pound ball discharged with a velocity of 1200 feet per second penetrates 34 inches into good dry oak, and a 24 pound ball with a velocity of 1300 feet penetrates 15 feet into dry earth.

If the obstacle be very thin, and the velocity of the ball very great, it may pass through without communicating any motion.
HUTTON, *ubi supra.*

194. From the experiments of Hutton and Robins practical inferences of great value have been deduced.

(1.) As great velocities are soon reduced within a certain limit, and as they depend upon the quantity of powder, the maximum service charge has been reduced to $\frac{2}{3}$ds of the weight of the ball:

Even this charge is not necessary except in the case of battering in breach, where the object is very near, and so thick that the ball lodges without passing through.

In close naval engagements great velocities are injurious, for the ball may then pass through both sides of the enemy's vessel without lodging, and the number of splinters produced by a ball in rapid motion is much less than is caused by one moving more slowly.

(2.) The charge being reduced, it is no longer considered necessary that the gun should be as strong; hence the thickness of cannon has been reduced, and the cheaper material of iron substituted for brass.

(3.) As the velocities increase only with the fifth roots of the lengths of bore, cannon have been much shortened without losing in effect. Eighteen calibers is now considered the maximum length of guns for the field service. In the naval service, carronades have been introduced; these are a short species of gun; their bore is smaller than in long guns of the same calibre, the shot must consequently be truly spherical; their charge is only one-twelfth of the weight of the ball; their weight and thickness are proportionably reduced; yet in close action, they produce effects superior to those of long guns.

The advantage of rifling the barrels of small arms, appears to be owing to the circumstance, that in this way a rotatory motion is given to the ball around the axis of the piece; thus the aim is more certain than with a simple cylindric barrel.

195. When a ball is projected from a piece of ordnance at a small angle of elevation, and falls upon water, or on a plane of hard earth, its flight will not cease, but it will rise again and perform a second trajectory, similar to its first, but less; and will continue to rebound until the whole of its projectile velocity is destroyed.

This species of motion, that resembles in some respects the reflection of elastic bodies, is called in Military Projectiles, the *Ricochet*.

Ricochet firing, which is executed at small angles of elevation, and generally with small velocities, is applied with great advantage from sea-coast batteries upon shipping, and in the attack of fortresses. It is to its use in the latter case, that the superiority of the art of attack over the most perfect systems of fortification, is to be attributed.

OF CENTRAL FORCES.

196. If a body be set in motion by two forces that are not in the same direction, and if one of them communicates a determinate velocity and then ceases to act, while the other is an accelerating force and continues to influence it without interruption, the path described by their joint action will be a curved line.

Of this general law, the instances already given of the motion of pendulums and projectiles are no more than particular cases.

The body moving in a curved line may be considered, at any instant whatever, as if it were moving along a tangent to the curve at the point where it then is; and if the accelerating force were to cease to act, the body would move on with uniform velocity in the direction of the tangent. If the accelerating force tend to solicit the body towards a point, this point is called the centre of force; and the force itself is called a centripetal force.

The centrifugal force is that by which it would tend to recede from the centre, were it not prevented by the centripetal force.

These two forces are called central forces.

The path, or trajectory, described by a body acted upon by central forces, is called its *orbit*.

The *Radius Vector*, is a line drawn from the centre of force to the point of the orbit at which the body is found.

The *Angular Velocity*, is the velocity with which the angle contained by any two positions of the radius vector is described.

The *Periodic Time*, is that which a body moving in a re-entering orbit employs after passing any given point, until it returns again to the same point. GREGORY, § 279.

197. If a body revolve in an orbit by the joint action of a projectile and centripetal force, the radius vector will describe equal areas in equal times, and in unequal times areas proportional to the times. NEWTON, § *Principia*, lib. 1. sect. 2.

Huygens, in investigating the laws that govern the motion of bodies acted upon by central forces, deduced them from the consideration of circular motions. This mode of proceeding has its simplicity to recommend it, and being elementary, has been made use of in this course. See HUGENIUS, § *de Vi Centrifuga.*

198. If a body move in a circle by the action of central forces, the arcs described in equal times are equal. POISSON, § 259.

199. If f be the measure of the central forces, v the velocity, and r the radius of the circle,

$$f = \frac{v^2}{r}$$ POISSON, § 259.

200. In order to compare central forces in a circle, with the attraction of gravitation, let v be the velocity due to the height h, then

$$\frac{f}{g} = \frac{2h}{r}$$

and if $2h = r$, the central force will be equivalent to that of gravity.

If then a body revolve in a circular orbit, and be acted upon by gravity, the projectile velocity must be equal to what would have been acquired in falling through half the radius.

Poisson, § 260.

201. The central forces of bodies revolving in circular orbits, are directly as the quantities of matter and as the radii of the orbits, inversely as the squares of the periodic times.

Poisson, § 261.

202. If the central forces in circular orbits be inversely as the squares of the radii, the squares of the periodic times are proportioned to the cubes of the radii.

The two last propositions may be extended to the case of bodies revolving in any orbit whatsoever, particularly in elliptic orbits. In the latter instance, half the greater axis of the ellipse is to be substituted for the radius of the circle.

If the centripetal force be gravity, its intensity diminishes as the squares of the distance increase; the above law ought therefore to hold good in the motion of the heavenly bodies, provided gravity be the force which retains them in their orbits.

203. Kepler, the astronomer, deduced the following laws of the motion of the planetary bodies from his own observations, and those of Tycho Brahé.

(1.) The planets move in plane curves, and their *radii vectores* describe around the centre of the sun, equal spaces in equal times.

(2.) The orbits of the planets are ellipses, of which the centre of the sun occupies one of the foci.

(3.) The squares of the times of the planets' revolutions are to one another as the cubes of the greater axes of their orbits

Subsequent observations have shown, that the focus is not occupied by the centre of magnitude of the sun, but by a point within his body, which is the common centre of gravity of the solar system. If the planets are acted upon by a mutual attraction whose intensity is inversely as the squares of the distances, the above laws are the same as might be deduced *a priori*. The attractive force is that of gravitation.

A farther discussion of this subject would encroach upon the field of *Physical Astronomy*.

204. When a solid body turns around a fixed axis, its several points describe circles, whose planes are perpendicular to, and their centres in, the axis ; the centrifugal force that acts upon any one point, is therefore as the radius of the circle which the point describes, and its direction is in the prolongation of the radius. The force of gravity is due to the attraction of the mass of the earth, but because the earth revolves upon its axis, the attractive force is lessened by the action of the centrifugal force, and thus at every part of the earth's surface, except at the poles, the measure of the force of gravity is less than it would have been had the earth remained at rest. Poisson, § 262.

At the equator, the centrifugal force and the attraction of the earth are directly opposed to each other; their resultant is therefore equal to their difference. At other parts of the surface of the earth these forces are oblique to each other.

205. If the earth were a perfect sphere, the centrifugal force at the equator would be to that at any other latitude, as the square of radius to the square of the cosine of the latitude. The centrifugal force at the equator, is to the whole attractive force of the earth, as 1 to 289. Poisson, § 263.

206. The greater intensity of the centrifugal force at the equator, has caused an increase of the equatorial diameter of the earth, and a consequent diminution of its polar axis ; the figure of the

earth is therefore more near that of an oblate spheroid than a sphere.

For this reason, the law of the decrease of the centrifugal force from the equator towards the pole, is not exactly what has been stated above; and the proportion of that force at the equator to the whole attractive force of the earth, becomes as great as 1 to 176. Poisson, § 264.

OF THE MOTION OF SYSTEMS OF BODIES.

207. The centre of gravity of a system of bodies, that has no fixed point, moves in space in the same manner that it would if the masses of the several bodies were united in it, and the forces that act upon the several bodies were applied to this centre, in directions parallel to their own, and without any change in their intensities. Poisson, § 458.

If the system have parts, that although not absolutely fixed, are compelled to move upon given curves or surfaces, the same law will still apply; but it will be necessary to include among the forces the resistances exercised by the curves or surfaces. As a corollary, we may deduce from the above proposition, that the motion of the centre of gravity of a system of bodies will be uniform and rectilineal, whenever no accelerating force affects the system, except the mutual action of the several bodies upon each other. This will be true, whether the mutual action be produced by an attractive or a repulsive force, or by actual collision. See §

208. If a number of bodies composing a system, and acted upon by a mutual attraction, but by no other accelerating force, be in motion, and if there be no fixed point in the system; the sums of the areas described around any given point, are proportioned to the times employed in describing them. Poisson, § 460.

If there be any fixed point in the system, the proposition is only true when this point is taken as the origin of the co-ordinates, and as the centre of the areas.

It has already been stated, that change of motion is never effected *per saltum*, but is always gradual; this principle is called the *Law of Continuity.*

209. Whenever the motion of a system of bodies undergoes any sudden change, the sum of the products that arise from multiplying each body into the square of its velocity, becomes less; and the diminution is equal to the sum of the products of each body into the squares of the velocities lost or gained by the moving bodies. Poisson, § 425.

In the case where the bodies that compose the system are free to move in any direction, or where they move in conformity with the law of continuity in known curves, or upon given surfaces, the velocity lost or gained at each instant, by each of the points, is infinitely small; hence the sum of the products of the bodies into the squares of the lost or gained velocities, in a given time, is also infinitely small. The sum of the products of each body, into the square of its velocity, is therefore constant when the bodies are not acted upon by any accelerating force.

210. If the accelerating force that acts upon a body be uniform, the distance to which the body will go before it acquires or loses a given velocity, will be as the square of that velocity.

On this theorem is founded that estimation of the force of moving bodies, which is known by the name of vis viva; and which makes that force proportional to the square of the velocity. This subject has already been referred to, in treating of the shock of elastic bodies. The truth is, that the effect of a body in motion, may be measured either by the distance to which it goes, or by the time that elapses before a resistance of uniform intensity reduce it to rest. If the effect be measured in the first of these ways, it will be found proportioned to the square of the velocity; in the second, to the velocity simply. Both these measures may be considered correct, and are perfectly consistent if rightly understood; but the same term cannot be applied to things so radically different. In the fundamental investigations of dynamics, we conceive the force to be proportioned to the velocity simply; but there are innumerable cases, particularly when investigating the action of machines, where the distance to which a body will go before a given resistance destroys its motion, becomes an object of inquiry. This distance then is the measure of a certain power residing in the moving body, that it has been proposed to distinguish by the term *Impetus*.

Playfair, § 99.

In the beginning of the eighteenth century, there was a warm discussion among philosophers upon this subject, but it is now well known to have been a mere dispute about words.

In addition to the above general properties of the motion of systems of bodies, there is another known by the name of the principle of least action; for an illustration of which, see Poisson, § 477. and 304.

MECHANICS.

THIRD SECTION—HYDROSTATICS.

OF FLUIDS.

211. A fluid is a body that yields to the slightest force impressed, and whose particles in yielding are easily moved among each other. NEWTON, *Principia*, lib. 2. § 5.

The nature of the particles of which fluids are composed is unknown. Some of the ancients believed that fluidity was caused by the particles being of a spherical figure; others held that the particles were in a state of continual intestine motion. Modern chemistry has shown that heat existing in a latent state is the cause of fluidity.

212. The fundamental and characteristic property of fluids, which distinguishes them from solid bodies, and serves as the basis of the theory of their equilibrium and motion, is that of transmitting pressure equally in all directions. POISSON, § 479.

213. In treating of the Mechanics of fluids, we distinguish them into two classes, elastic and non-elastic, or compressible and incompressible.

Non-elastic fluids are also called liquids.

Although writers on the theory of Mechanics consider liquids as incompressible, they are not absolutely so.

This might have been inferred from the well-known fact that their volumes are augmented by heat and diminished by cold. The philosophers of the continent of Europe, misled by the famous Florentine experiment, have notwithstanding, until lately, asserted that they could not be compressed. Mr. Canton, an English philosopher, has, however, shown, that water, a fluid of this class, is compressible to a certain extent. See PHILOSOPHICAL TRANSACTIONS ABRIDGED, Vol. XI. and XII.

This has been more fully proved by our countryman, Mr. Perkins. SILLIMAN's JOURNAL, Vol. 3.

Elastic fluids are either permanently so, or condensible; the former are called *gases* or aeriform fluids, the latter go by the name of vapour or steam.

214. If a fluid, considered as devoid of might, be contained in a vessel, and if pressure be applied by the intervention of a piston passing through an orifice in the vessel, that pressure will be equally distributed through the fluid in every possible direction. POISSON, § 479.

This pressure is not only distributed equally through the fluid in all directions, but it acts perpendicularly against every point of the vessel that contains the fluid.

A non-elastic fluid may therefore be considered as a machine; for a machine is, by definition, an instrument, by means of which a force is made to act upon points that are not in its direction, and to exert upon them a greater or less effort than if it were directly applied. POISSON, § 480.

215. If a fluid be contained in an open vessel, and remain in equilibrio notwithstanding, it must be solicited by forces that are perpendicular to its surface.

The directions of the force of gravity may be considered as parallel to each other within limited distances, and consequently the surface of a gravitating fluid contained in a vessel, in order to be equilibrio, must be a horizontal plane.

If the extent of surface be great, it assumes a spheroidal figure, corresponding with, or parallel to, the figure of the earth.

The surface of a gravitating fluid in equilibrio is said to be level.

216. If a liquid be placed in a number of vessels communicating with each other at bottom, it will not be in equilibrio unless all the surfaces be in the same horizontal plane. Water, therefore, rises to equal heights in the opposite arms of a bent tube. POISSON, § 508, 509.

217. Levelling is the art of drawing a line near the surface of the earth, that shall cut the directions of gravity every where at right angles. PLAYFAIR, § 243.

A level line is nearly an arc of a circle, but as the instruments called levels do not give this line, but its tangents only, a reduction is necessary wherever the distance exceeds a certain limit. If L be the length of the arc in English miles, and D its depression in feet below one extremity of the tangent drawn to it at the other extremity:

$$D = \frac{2\,L^2}{3}.$$

There are three different kinds of level in common use:

(1) The Water level; in this a fluid is placed in an open tube bent twice at right angles; the sight directed over the surfaces in the two perpendicular branches, gives a tangent to the level line.

(2) The Spirit level consists of a tube hermetically sealed at both ends, enclosing spirit of wine and a small bubble of air. As the air has a tendency to rise to the highest part of the tube, it is inferred that the axis is horizontal when the bubble remains at rest in the middle of the tube; if sights be adapted to this in such a way that the line joining them shall be parallel to the axis of the tube, this line will be a tangent to the level line when the bubble is in the position we have just mentioned.

(3) In the Mason's level, the direction of gravity is pointed out by the plumb line, and a perpendicular to the plumb line is of course a tangent to the level line.

A modification of this instrument, called the American level, has been sometimes used in laying out the directions of canals. It consists of two laths joined to each other, like the sides of an acute angled isosceles triangle; the plumb line is suspended from the vertex. See PUISSANT, § *Nievellement.* REES' CYCLOPÆDIA, art. *Levelling*

OF THE PRESSURE OF FLUIDS.

218. The pressure of a gravitating fluid upon a horizontal base is equal to the weight of a column of the fluid resting upon the same base, and whose altitude is equal to the perpendicular height of the surface of the fluids, above the base.

The pressure on a horizontal plane is therefore proportioned to the base and to the depth of the fluid; whatever be the figure of the vessel in which it is contained, or the extent of the fluid in a horizontal direction.

If three vessels be taken whose bases are equal, if one be perfectly cylindrical, another narrowing towards the top, and the third wider at top than at bottom; and if these three vessels be filled with water, or some other liquid, to the same depth, the pressure upon the three bases will be exactly equal. This result is fully proved by experiment. Poisson, § 493.

The best apparatus for performing this experiment goes by the name of the vases of Pascal. See *Description d'un Cabinet de Physique.*

If the depths of the same fluid, contained in different vessels, be equal, the pressure is as the bases. If the bases be equal, the pressure is as the depths.

219. If one of the sides of the vessel be a plane that is not horizontal, the pressure upon it is proportioned to its area, and to the distance of its centre of gravity below the surface of the fluid. Poisson, § 495.

The proper thickness of walls of a material of known strength, to resist the pressure of a fluid of given depth, may be determined upon this principle. See Belidor, *Architecture Hydralique.* Prony. Gauthey, *Memoires*, vol. 3.

The *Centre of Pressure* is the point through which the resultant of all the pressures on a surface passes, or to which, if a force equal to the whole pressure were applied in a contrary direction, the surface would remain at rest.

If all points in the surface were pressed by equal forces, the centre of pressure would coincide with the centre of gravity; but as the pressure of a gravitating fluid augments with the depth, the position of the centre of pressure is always lower than that of the centre of gravity.

220. Because fluids press in proportion to their altitudes and not their quantities, it follows that any quantity of fluid, however small, may be made to balance any other quantity, however great.

This principle has been called the Hydrostatic paradox, and an apparatus called the Hydrostatic bellows has been contrived for illustrating it.

Upon this principle also has been constructed an apparatus for pressure, called after the name of the inventor, *Bramah's Press.* This will be described when we come to treat of Hydraulic Engines.

221. If two immiscible fluids are in equilibrio in the opposite arms of a bent tube, their heights above the surface of contact will be inversely as the densities of the fluids. POISSON, § 509.

Density has been defined as the ratio between the weights of equal bulks of different bodies.

222. If a solid body of less density than a liquid be placed upon the liquid, it will sink until it has displaced as much of the fluid as is equal to its own weight, and will then float at the surface.

223. A solid body immersed in a fluid will, when left to itself, sink, if its density be greater than that of the fluid; rise to the surface, and float there, if its density be less; or remain at rest, if its density be equal. POISSON, § 501.

The force with which a body rises or sinks in a fluid, is equal to the difference between the weight of the body and the weight of an equal bulk of the fluid. Hence, if a body be weighed in a fluid, it will lose in weight as much as the weight of an equal bulk of the fluid.

It thus happens that the weight of a fluid contained in a vessel is not sensible while the vessel is immersed in a mass of the same fluid, but becomes evident the moment we attempt to raise the vessel above the surface. From a misapprehension of this experiment, the ancient philosophers asserted that fluids had no weight *in proprio loco;* They supposed the weight to be destroyed, when in fact it is merely supported. Fluids appear devoid of weight when placed in fluids of the same description, not because they do not still gravitate, but because a hydrostatic pressure acts upon them that is equal to their gravity, and in a contrary direction.

224. If the same body float upon the surfaces of two different fluids, the respective bulks of the parts immersed will be inversely as the densities of the fluids; and if a body sink in two different fluids, the weight it loses will be directly as the densities of the fluids.

OF SPECIFIC GRAVITIES.

225. Specific Gravity is the proportion which the weight of a body bears to the weight of an equal bulk of another body.

In this general sense it is the same with density; but it is usual to compare the specific gravity of bodies by means of some common standard, that serves as the unit; and in terms of which the specific gravities of all other bodies are expressed. The standard universally adopted in the specific gravities of liquid and solid bodies is pure water; and this may be obtained by means of distillation, or by making use of rain water newly fallen in places where it is not liable to be contaminated in passing through the atmosphere.

226. It is a general law of nature, that bodies are expanded by heat, and contracted by cold; the bulk, therefore, both of the standard and the bodies to be compared with it will be affected by change of temperature, and their specific gravities will in consequence be changed.

For this reason it is necessary to adopt some given temperature at which experiments shall be made, or to which the results shall be reduced. The French Institute have adopted the point of the greatest density of water, which is from 38° to 40° of Fahrenheit's thermometer. The tables of the English philosophers are constructed for the temperature of 60° of Fahrenheit.

227. From the definition of Density, the following relations between the weights, bulks, and specific gravities of bodies, may be deduced.

(1) If the weights be equal, the specific gravities are inversely as the bulks.

(2) If the bulks be equal, the specific gravities are directly as the weights.

(3) If the specific gravities be equal, the weights are directly as the bulks.

We owe the invention of the method of Specific Gravities to Archimedes. He observed that a body immersed in a vessel containing a fluid, causes the fluid to rise; he thence inferred that the rise was equal to the bulk of the body immersed; and that if two bodies of equal weights, but unequal densities, were plunged in the fluid, the rise would be inversely as their densities. He applied this principle to ascertain whether a crown of gold, made for Hiero, king of Syracuse, was adulterated or not. See PLUTARCH, *in Archimedem*.

228. The difference between the absolute weight of a body, and its weight when entirely immersed in a fluid, is equal to the weight of an equal bulk of the fluid; and when the bulks of bodies are equal, their specific gravities are directly as their weights. If then s be the specific gravity of the fluid, S that of the solid body, W the absolute weight of the latter, and W′ its weight in water:

$$S : s :: W : W - W', \text{ and}$$

$$S = \frac{sW'}{W - W'}$$

but if $s = 1$, which is the case when the fluid employed is the standard:

$$S = \frac{W}{W - W'}$$

The weight of bodies as determined in the open air, is in some measure affected by the action of the atmosphere, and the specific gravity in consequence; but this it is usual to neglect except in very accurate investigations.

229. The instrument now chiefly employed in determining the specific gravities of bodies proceeds upon the above principle; it is called the Hydrostatic balance and is no more than a very accurate balance, with such additions as will enable the experimenter to weigh bodies with facility in air and water alternately.

230. The general rule for finding specific gravities by means of the Hydrostatic balance, is: weigh the body first in air, and then in water, divide the weight in air by the difference between the weight in air and the weight in water, the quotient is the proximate specific gravity. This rule is liable to modifications in its application, arising from the nature of the substances under experiment.

(1) If the substance be a solid body heavier than water and in one solid mass, it may first be weighed in one of the scales of the balance, and then suspended from beneath it in a vessel of water, when it is again weighed. The body must not only possess the above requisites, but be insoluble in water. If it be soluble in water, it may be weighed in a fluid that does not act upon it, and whose specific gravity, compared with water, is known.

(2) In the case of powders that cannot be tied and suspended to the scale; they are placed in a bucket whose weights in air and in water are known, and thus weighed alternately in air and in water.

(3) If the body be a solid lighter than water, it must be weighed in air, and then attached to a body so heavy as to cause it to sink in water; the compound must then be weighed in water, and after-

wards the heavier body; if now the weight of the light body in air be divided by the difference between the several losses of weight of the compound and of the heavier body alone, the quotient will be the specific gravity.

(4) To find the specific gravity of a liquid, take a phial whose weight when empty and when filled with pure water to a certain mark is known; fill it with the liquid up to the same mark, and weigh it. In this manner you will have the weights of equal bulks of the liquid and of water; divide the first of these by the last, the quotient is the specific gravity. The same method may be employed to ascertain the specific gravity of pulverulent bodies that are soluble in water.

The specific gravity of a liquid may also be obtained by weighing the same solid in air, in water, and in the liquid; the loss of weight of the solid when weighed in the liquid, divided by the loss of weight in water, gives the specific gravity.

231. When the experiments are made at a temperature different from that chosen as the standard, a correction must be applied. This consists in multiplying the specific gravity found by the above methods by the specific gravity of water at the given temperature. To facilitate this operation, accurate tables have been constructed of the density of water at all the temperatures within which experiments are made.

For such a table, see **Brande's** *Chemistry*, § 488. and **Biot**, *Traite de Physique*, Vol. I. p. 425.

232. In very accurate investigations, a correction becomes necessary for the loss of weight sustained in consequence of the bodies being weighed in air, and not in vacuo. If s be the specific gravity of the body, as ascertained by weighing it in air and water, m the specific gravity of air at the time of making the experiment, S the absolute specific gravity:

$$S = s + m(1 - s)$$

When the body is heavier than water this correction is to be subtracted; when lighter it is to be added. In the mean state of the atmosphere $m = .00122$ nearly, water being the unit. **Playfair**, § 251, 252.

The method of finding the specific gravity of air and gases will be afterwards explained.

233. A knowledge of the specific gravities of bodies is of great importance in the physical sciences, in many of the arts, and in commerce.

(1) We can, by ascertaining their specific gravities, determine the difference between two bodies very similar in external appearance, and thus discover their nature, where chemical analysis would be inapplicable.

(2) We are furnished with infallible means of detecting adulterations in drugs, &c. of knowing the value and activity of chemical preparations, and the strength of saline solutions.

(3) We may in this way judge of the purity and value of coin, particularly of gold coin, the richness of metallic ores, &c.

(4) One very important application of specific gravities is to ascertain the strength of spiritous liquors. This cannot be determined as well in any other way.

234. The method of specific gravities furnishes us with the means of determining the bulks of bodies whose specific gravities are known, and of calculating the magnitude of irregular solids.

If B be the bulk of a body in English cubic inches, W its weight in grains, and S its specific gravity:

$$B = \frac{W}{252.576\ S}$$

Where the weight is in Troy ounces:

$$B = \frac{W}{.52746\ S}$$

PLAYFAIR, § 253.

When the bulk is to be determined in cubic feet, and the weight is given in avoirdupois ounces:

$$B = \frac{W}{1000\ S}$$

The above theorems are founded upon the well-known facts that a cubic inch of pure water, at the temperature of 60° of Fahrenheit, weighs 252.576 grs. English Troy weight, and a cubic foot of pure water 1000 oz. avoirdupois.

From the above equations the manner of finding the weight of a body whose specific gravity and magnitude are known may be at once deduced.

OF THE HYDROMETER.

235. The Hydrometer is an instrument for determining the specific gravities of fluids; its theory depends upon the principle already laid down, § 224. That if a body float alternately upon two different fluids, the magnitudes of the parts immersed will be inversely as their specific gravities. The essential parts of the Hydrometer are a bulb or hollow ball, a stem on which a scale is drawn, and a weight by which the instrument is retained in a perpendicular position, and made to sink to a determinate depth in some given fluid.

The common Hydrometer is made of glass; the weight is a second bulb, filled with a proper quantity of mercury or leaden shot. It is, from the brittleness of the material, very liable to accidents; and as the stem must necessarily be slender, when compared with the bulb, the scope of the scale is limited. To determine the various specific gravities of all liquids, whether lighter or heavier than water, a single hydrometer will not suffice. On this account it is not a convenient instrument for general use. On the other hand, as glass is acted upon by but few fluids, it is a valuable instrument for chemical purposes. In consequence of the brittleness of glass, metal has been substituted for it in the construction of hydrometers, and the scope of the scale has been increased by means of moveable weights.

Dicas' Hydrometer is used in the customs of the United States; it is made of silver gilt, has 36 weights, and is capable of measuring all specific gravities from 1 to .825. In the hands of the chemist it would be an instrument of value for nice experiments, but it is not well fitted for the use to which it is applied. One better adapted for ordinary purposes is the Hydrometer of Quin; it is of brass, and although it has no more than four weights, it will determine specific gravities within the same limits as that of Dicas.

Southworth, an ingenious artist of New-York, has constructed one that promises to supersede Dicas' in the customs, and with great advantage. It is made of silver, and has but one weight. Its scale is not less extensive than Dicas' or Quin's.

The Hydrometer is of great use in pharmaceutical preparations, and in the manufacture of sugar and of salts, by determining the exact time at which the process of evaporation by heat is to be stopped.

A variety of it employed to detect the presence of sugar in the processes of the brewer and distiller is called the *Saccharometer.*

Another variety is used to discover adulterations in milk ; it is called the *Lactometer.*

236. The Hydrometer is besides applied to weigh and find the specific gravity of solids. The Hydrometer adapted for this purpose is the invention of Mr. Nicholson, and called after his name. See NICHOLSON, *Philosophy*, p. 303. GREGORY, *Mechanics* § 404.

237. Specific Gravities determined by the Hydrometer, are, like those formed by the Hydrostatic balance, affected by change of temperature ; a thermometer should therefore be always employed and examined at the same time with the Hydrometer.

In spiritous liquors, the difference between the apparent specific gravity and the true, caused by change of temperature, is very great. To facilitate the reduction of the first of these to the last, tables have been constructed from accurate experiments. See CAVALLO, *Philosophy*, Vol. II.

Dicas' Hydrometer is accompanied with a sliding rule, by which this reduction is rendered very easy.

238. When alcohol and water are mixed together, the compound is not found to possess the same magnitude of the two ingredients, but is less ; this diminution in bulk is called concentration. It takes place not only between these two substances, but in most other cases of the mixture of bodies, whether solid or fluid.

For this reason the famous rule of Archimedes, for determining the proportion of two known ingredients from the specific gravity of their compound, is of no value in practice.

239. The specific gravity of fluids may be ascertained in another way, by means of a set of hollow bubbles of glass of known specific gravities. See Parke, *Chemical Essays*, Vol. II.

OF THE ATMOSPHERE.

240. We discover, while investigating the properties of fluids, that we are continually immersed in a body of that class. This is the *Air* or Atmosphere that surrounds our globe. The atmosphere being insipid, inodorous, and invisible, its material nature long escaped the attention of philosophers.

The first proof of its materiality may be derived from the action of its currents, or the winds. These are capable of producing considerable effects.

Air also prevents water, or any other liquid, from entering into a close vessel containing it, when the vessel is inverted and plunged into the fluid; it is consequently impenetrable.

241. If a tube, open at both ends, have one of its extremities immersed in a vessel of mercury, and if a piston, fitted air tight into the tube, have previously been depressed to the end immersed; on drawing the piston up, the mercury will follow it to the height of about 30 inches. If the piston be raised to a greater height, the mercury no longer follows it, but its surface remains at rest at the height of 30 inches. If a similar experiment be performed with water instead of mercury, the water will follow the piston to the height of 34 feet, and there stop. Deluc, *Recherches sur l'Atmosphere.*

The ancient philosophers attributed the rise of water in tubes, to the abhorrence they supposed nature to entertain of a vacuum. But it is now known to be caused by the pressure of the atmosphere. The limit to the rise of water was observed and communicated to Galileo by some workmen employed in the service of the Grand Duke of Tuscany. Torricelli, his pupil, imagined the substitution of mercury for water, in the experiment, in the following manner, viz.

242. If a tube upwards of 30 inches in length, hermetically sealed at one end, and open at the other, be filled with mercury, and carefully inverted into a basin of the same liquid, the mercury will fall in the tube until its surface stands at about 30 inches above the level of the mercury in the basin. At this level it will come to rest. DELUC, § 18.

This instrument was at first called the Torricellian Tube, it is now named the Barometer. The space left in the upper part of the tube is called the Torricellian Vacuum.

243. As the proportion between 30 inches and 34 feet, is inversely that of the specific gravities of the respective fluids, and as this is the same with the relative quantity of each other, they would mutually counterbalance ; we infer that the force which sustains them in the tubes is the same.

If this force be the pressure of the atmosphere acting in consequence of its gravity, it is manifest that the mercury in the Torricellian Tube must stand highest near the surface of the earth, and that its altitude must diminish as it is raised in the atmosphere.

An experiment of this nature was planned by Pascal, and executed by Perrier, at the Mountain called the Puy de Dome, in the French province of Auvergne. See PASCAL, *Œuvres*, Vol. 4. EDINBURGH REVIEW, Vol. 20. DELUC, § 14.

Of the Pump.

244. Water rises in the common pump by the action of the pressure of the atmosphere. A common pump is composed of a cylinder containing two valves or *boxes*, opening upwards ; the lower valve is fixed in the barrel of the pump, the upper in a piston that works air tight in the barrel ; by alternately raising and depressing the piston; the air is drawn out of the

barrel, and the water rises to supply its place ; when all the air is drawn out the water rises to the upper valve, passes through it at the moment of depression, and is then lifted upwards to the spout, or place of discharge.

This species of pump will not raise water above 34 feet.

There are two other kinds of pump, the forcing and the lifting ; but the consideration of their several characteristics must be deferred for the present.

Of the Syphon.

245. If a bent tube, open at both ends, have the end of the shorter leg immersed in water ; and if the air in the tube be sucked out, or otherwise extracted, the water will flow through the tube in a continued stream, provided the height of the upper part of the tube above the surface of the water do not exceed 34 feet. PLAYFAIR, § 343.

A bent tube of this kind is called a *syphon.*

The liquid is supported in the tube by the pressure of the air, and runs with a velocity due to a depth of fluid equal to the difference of altitude between the levels of the surface of the fluid in the vessel and of the discharging end of the tube.

It is not absolutely necessary that the shorter leg should be immersed ; for if the long leg of the syphon be immersed to a depth that will bring the orifice at the other extremity beneath the level of the fluid ; it will still discharge it, until the surface sink to a level with the opening.

The syphon is much used for decanting liquors, and is commonly called a *Crane.*

An amusing philosophical plaything is sometimes made by placing a syphon within a figure in a cup. As soon as the liquor rises within it to a certain height, the syphon is filled, the discharge commences, and continues until it be all exhausted. This apparatus is called Tantalus' cup.

OF THE PRESSURE OF THE ATMOSPHERE.

246. The mean height at which mercury is sustained in the barometer at the level of the sea, is about thirty inches. The force that supports it is the pressure of the air; the weight of a column of mercury is therefore the measure of the pressure of the atmosphere upon a surface equal to the area of the base of the column. If the specific gravity of water be taken as 1000, that of air is $1\frac{2}{9}$, and that of mercury 13600; from these data, the pressure of the atmosphere at the earth's surface appears to be about 15 lbs. per square inch; and it is equivalent to that of a homogeneous atmosphere, 27600 feet in altitude.

The pressure being 15 lbs. per square inch, a middle-sized man is loaded with a weight of about eleven and a half tons. We are insensible to this enormous burthen, because it presses equally in all directions, and because the air is so perfect a fluid that its particles offer no interruption to our motion through it.

If the air were a non-elastic fluid, and the atmosphere consequently homogeneous, its height would, upon the above hypothesis, be 27600 feet. We shall find occasion to make use of this estimate more than once in our subsequent investigations, although, in consequence of the elasticity of the air, it is far from the truth.

OF THE AIR PUMP.

The mechanical properties of the air are best investigated by the *Air Pump*.

247. The Air Pump is a machine by means of which the air may be extracted from close vessels. It is composed, essentially, of a barrel and two valves like those of the common pump; the barrel communicates with a close vessel, called a receiver, and as the piston alternately rises and falls, the air in the receiver is rarefied.

The air pump was invented by Otto Guericke, a burgomaster of Magdeburg in Germany, in the year 1664. GUERICKE, *Experimenta Nova de vacuo spatio.*

It has successively received many great improvements, particularly from Boyle, Hawksbee, Martin, Smeaton, Cuthbertson, Mendleshom, and Dumoutier.

By combining two barrels together, and working the pistons with rack work turned by a toothed wheel, the operation has been rendered more rapid. By adapting a contrivance that opens the lower valve at each stroke of the piston, the former limit of the rarefaction, which ceased as soon as the spring of the contained air became too weak to open this valve, may now be passed.

The pumps of Boyle and Hawksbee, and many of the later ones, had plates of brass covered with oiled leather on which the receiver was set. In pumps of this construction, an evaporation took place from the surface of the leather that affected the results of experiment; in the best modern pumps, the plate and the bottom of the receiver are ground to fit each other.

The barrels that were formerly of brass, are now made of glass, and the pistons that were also of brass and covered with oiled leather, are made of a soft metal that is forced into the barrel. GREGORY, *Mechanics*, Vol. II.

To measure the exhaustion produced by an air pump, we make use of gauges of several different kinds.

(1) The Barometer gauge; this is a tube open at both ends; the upper communicating with the receiver, the lower plunged in a basin of mercury; as the air is exhausted from the receiver, the mercury will rise in the tube, and the ratio its height bears to that of the mercury in a barometer will mark the rarefaction the pump produces.

(2) The short Barometer gauge; this is a short tube hermetically sealed at top, filled with mercury, and inverted in a vessel of that fluid, the surface of which is enclosed in a receiver communicating with the valves of the pump; when the pump is in operation, the air in this receiver is exhausted in equal measure with that in the principal one,

and when the pressure becomes less than that of a column of mercury of the length of the tube, it will of course begin to descend.

(3) The Syphon gauge resembles in principle the short barometer gauge, but is turned up at the end instead of being inverted in mercury.

(4) The Pear gauge; this is a hollow vessel of glass that takes its name from its shape; it has a small orifice at its larger end, which is immersed in a vessel of mercury; it is thus placed under a separate receiver communicating with the valves of the pump; when the receivers are exhausted, the air contained in the gauge expands itself, and forces its way through the mercury, but when the air of the atmosphere is re-admitted, it forces the mercury into the gauge until the rarefied air left in it is brought to the same density with the atmosphere; the space unoccupied by mercury will therefore be the measure of the rarefaction of the air in the receivers.

In using this instrument in a common air pump, at the same time with the barometer gauge, there is a remarkable discrepancy observed between their respective results; the pear gauge showing a degree of rarefaction far beyond what is pointed out by the other. This is readily explained, when we consider that the oil in the leather that covers the plate of the common pump will evaporate when the pressure of the atmosphere is removed, and fill the cylinder with a rare vapour, which, being elastic, tends to prevent the rise of the mercury in the barometer gauge; no part of this vapour can enter the pear gauge, which is therefore insensible to the existence of it in the receiver. See ENCYCLOPÆDIA BRITANNICA, article *Pneumatics;* and HACHETTE, *Cours Elementaire.*

248. Among the many proofs furnished by the air pump of the pressure of the atmosphere, are the following, viz.

(1) If the hand be placed on the top of an open receiver, it will be pressed down upon it with great violence after a few strokes of the pump.

(2) A receiver is firmly fixed to the plate of the air pump.

(3) A less receiver, placed within a greater, is fixed to the plate of the pump on the re-admission of the air.

(4) A bladder, stretched over the mouth of an open receiver, is burst with great noise.

(5) Two hemispheres of brass, ground to fit each other, are so firmly fastened together as to require a great effort to separate them. These hemispheres are called the Magdeburg hemispheres.

The last experiment might, if the exhaustion were perfect, furnish a mode of estimating the quantity of the atmospheric pressure in terms of some known weight and measure; but this may be better ascertained upon the principle already explained. See GUERICKE, *Experimenta Nova.*

249. The pressure of the atmosphere might reasonably be attributed as a cause to its being acted upon by gravity; but to render the fact of the weight and consequent gravity of atmospheric air more certain, a portion of it may be actually weighed; this is effected by adapting a valve opening outwards to a flask; the flask, after being weighed, is screwed upon the plate of an air pump, and the air rarefied as much as possible; if it be now weighed a second time it will be found lighter; the difference in weight is that of the air abstracted.

That air gravitates may also be made manifest from the fact, that the atmosphere continues to accompany the earth in its rapid diurnal and annual revolutions.

The strata of air at the surface of the earth are pressed by the whole weight of the superincumbent strata, and the height of the mercury in the barometer is the measure of this pressure. BIOT, Vol. I.

OF THE ELASTICITY OF AIR.

250. That air is elastic may be proved by many experiments, among which are the following, viz.

(1) If a bladder only partly filled with air be placed under the receiver of an air pump, and the air exhausted from the receiver, the bladder will distend itself, and finally burst.

(2) If a bladder, under similar circumstances, be loaded with heavy weights, on exhausting the air from the receiver, that within the bladder acquires force sufficient to raise these weights.

(3) The elasticity of the atmosphere is also shown by the manner in which the air pump itself works; for it would be impossible to exhaust the air by means of that machine, were it not elastic.

(4) The elasticity of the air may be likewise proved by merely inverting a close vessel full of air in water: the resistance it offers to farther immersion, and the partial entrance of the water into it, are proofs of the elasticity of the air it contains.

251. In consequence of the elasticity of air, we can at once infer, that if it were pressed by a weight in addition to that of the superincumbent strata of the atmosphere, it must occupy a less space, while, if the pressure of the atmosphere were lessened, it must dilate itself, and occupy a greater. But the exact law of condensation, and the relation between the space occupied by a given quantity of air, and the pressure by which it is acted upon, remain to be investigated. This may be done by means of the following experiments, viz.

(1) If an open tube be screwed into the top of a vessel partly filled with mercury, and if it dip into the mercury in such a way that the air contained in the upper part of the vessel shall be entirely enclosed; and if the whole be placed beneath a tall receiver upon the plate of an air pump; the mercury in the vessel will be forced up into the tube when the air is exhausted from the receiver, and it will be found to rise *pari passu*, and to equal heights with the mercury of the long barometer gauge. We may hence conclude that the elasticity of air of equal or less density than the air of the atmosphere, is exactly equivalent to its pressure.

(2) Take a cylindrical glass tube, bent into the form of a syphon, open at the extremity of the longer branch, and closed at the other; pour in at first a small quantity of mercury sufficient to fill the bent part of the tube, and thus enclose the air in the shorter branch without compressing it; pour in next at the open end an additional quantity of mercury, it will compress the air in the shorter branch, and rise into it, but not as fast as it rises in the open branch; and when the air enclosed in the shorter branch shall have thus been condensed into half the space it had formerly occupied, it will be found that the mercury in the open branch will stand as high above the common plane of the mercury and air as the barometer stands at the moment. In this case the original pressure is evidently doubled. When the altitude

of the mercury shall have reached to twice that of the barometer, at the instant the original pressure is tripled, and the enclosed air will be found to occupy exactly one third of the original space. Biot, *Traite de Physique*, Vol. I.

In order that the air contained may be perfectly dry, it is usual to invert the tube, and insert the open extremity through a cork into a receiver containing deliquescent salts for some time before the experiment is performed; by adapting the tube in the same way to a receiver, and filling it with gas, we may make experiments on the force of the gases.

(3) Take a very slender tube of glass, hermetically sealed at one end; introduce into it a small column of mercury; this column in a small tube will not permit the air to pass it, and if the sealed end of the tube be held downwards, it will compress the contained air by its weight; on the other hand, if the open end of the tube be held downwards, the mercury will descend a certain distance, and there stop. The quantity of compression in the one case, and of dilatation in the other, will furnish means of computing the relation between the pressures and the corresponding spaces. Biot, Vol. II.

(4) If a portion of air be left in the tube of a barometer, and the space it occupies observed before it is inverted in a basin of mercury, this space will be found to be very considerably increased after the inversion of the tube is effected; if the tube be inclined in various degrees, the space occupied by the air will be found to lessen until the tube be brought into a horizontal position, when the air will be of the same bulk as it was originally. Biot, Vol. II.

The second and fourth of these experiments were contrived by Mariotte, the third by Dalton. They fully confirm the following law, called, after the inventor, that of *Mariotte*.

252. The air of the atmosphere resists the action of forces that tend to compress it, with an elasticity that is directly as its density, and inversely as the space it occupies.

This law is also true of all other permanently elastic fluids.

In all the above experiments and deductions, the temperature is supposed to remain unchanged.

253. Air resists pressure equally in all directions, and if all pressure were removed, there is no assignable limit to the space a given quantity of air would occupy. PLAYFAIR, § 333.

254. It is usual in performing a course of experiments with the air pump, to show the necessity of the presence of air for the support of animal and vegetable life, and of combustion.

(1) An animal under the receiver of an air pump, becomes restless and uneasy as the air is exhausted, then falls into convulsions, and finally dies.

(2) A plant kept for some days in an exhausted receiver withers and dies.

(3) A candle is extinguished when the air is drawn from the receiver under which it is placed.

These experiments are no longer of much value, in consequence of the great light that has been thrown upon these subjects in another way by modern chemistry.

OF THE BAROMETER.

255. When the basin and tube of the Torricellian apparatus are firmly fastened to a common support, on which a scale is drawn, it forms the instrument known by the name of the barometer.

The altitude of the mercury in the barometer is reckoned by us in inches and decimals, and by the French in decimal parts of the metre.

The barometer, besides its original purpose of showing the pressure of the atmosphere, is now applied to the mensuration of heights, and to predict changes in the weather.

256. For both of these purposes, it was at one time considered essential that the scale of the barometer should be lengthen-

ed; to accomplish this, a number of changes were made at different times in its form; nearly all of these have been laid aside, in consequence of their being much more remarkable for ingenuity of contrivance, than for their practical utility.

The wheel barometer of Hooke is the only one that remains in common use; Professor Leslie of Edinburgh has proposed to restore the use of the conical or pendent barometer as a portable one.

The lengthening of the scale of the barometer has been rendered unnecessary, by the great improvement that has taken place of late years in the graduation of instruments; barometers are now constructed with terniers, that give the height of mercury to the nearest thousandth part of an inch.

257. When the barometer is used in the mensuration of heights it must be rendered portable; this can be effected in a variety of ways.

(1) The mercury may be enclosed in a leather bag or vessel whose softness and pliability permit the action of the air; the capacity of this vessel may be diminished by the pressure of a screw upon its lower extremity, and the mercury forced up until it fills the tube, and is held firmly without the power of oscillation.

(2) In Deluc's barometer, the general form resembles a syphon, but the upper part of the open branch is a separate tube of glass, joined to the other by a joint of ivory, in which a stop cock is inserted. By inclining the instrument the mercury is brought to strike against the top of the tube, and is fixed firmly in that position by closing the stop cock. In using this barometer it is necessary to employ a scale upon each branch of the syphon. Deluc, *sur l'Atmosphere.*

(3) A very complete portable barometer was planned by Sir George Sharpsburgh, and subsequently improved by Ramsden and Troughton. This instrument, when in use, is suspended by gembals from a tripod stand; its own weight is then sufficient to keep it steady when exposed to the weather. The legs are hollowed out, and, when put together, form a case, in which the tube, cistern, &c. are packed. See Rees' Cyclopædia, article *Barometer.*

(4) Another, still more portable, has been planned by Sir H. Englefield. It, with all its appendages, is enclosed in the space of a walking cane.

Measurement of Altitudes by the Barometer.

258. As we ascend from the surface of the earth the density of the air diminishes, for each stratum of air is compressed only by the weight of those above it; the upper strata are of course less compressed, and consequently less dense than those below them. Playfair, § 334.

Were the atmosphere homogeneous, the height of the mercury in the barometer would decrease in the same ratio with the distance from the surface; but as this is not the case, some other law remains to be investigated. This is done upon the hypothesis that the temperature and the force of gravitation are uniform throughout; and the following result is obtained.

259. If the heights from the surface be taken, increasing in arithmetical progression, the densities of the corresponding strata of air will decrease in geometrical progression; and if the heights from the surface be taken in arithmetical progression, the corresponding columns of mercury will decrease in geometric progression. Playfair, § 336.

As logarithms have to their numbers a relation, the converse of the above, it is evident that they might be employed in this calculation. If b be the height of the column of mercury at the surface, b' its height at any altitude, h above the surface, and m a constant co-efficient, to be determined by experiment:

$$h = m\,(\log. b - \log. b')$$

m may be determined by ascertaining the value of h, in some particular case where b and b' are known, by levelling or by trigonometric calculation. If the temperature be 32° of Fahrenheit's thermometer; if b and b' be taken in English inches, and h in English fathoms, $m = 10{,}000$; if then the difference between the tabular logarithms of the heights of mercury be taken, and if the mean temperature be that of freezing, it will be only necessary to remove the decimal point four places back, and it will give the height in fathoms.

260. This rule is true only in this one particular case; for the density of the air is affected by its temperature, and the column

$\frac{244}{100000} / 244000 = \frac{61}{25000} = \frac{1}{417}$

of mercury in the Barometer will be lengthened by heat, and contracted by cold; corrections must therefore be applied to the result of the above rule, for both these causes of irregularity.

Air has been found by the experiments of Sir Geo. Shucksburg and Gen. Roy, to expand .00244 of its own bulk for each degree of Fahrenheit's thermometer. The expansion of mercury for each degree of Fahrenheit's thermometer, is about .000101 of its bulk. See REES' CYCLOPÆDIA, article *Barometer*.

261. If, therefore, b be the height of the mercury at the lowest station, and b' at the highest; t and t' the temperatures of the air at these stations, q and q' the temperatures of the quicksilver in the two barometers; h the perpendicular height in fathoms:

$$h = 10.000. \left(1 + .00244\left(\frac{t+t'}{2} - 32^\circ\right)\right) \log. \frac{b}{b'(1 + .000101\,(q\,q')}$$

262. The following are the rules for the practical application of this problem, viz.

(1) In order that the observations may be carefully and accurately made, two portable barometers must be procured of the best construction, and filled with mercury of the same specific gravity; into the wooden frame of each, must be sunk a thermometer called its *attached* thermometer; two other thermometers must also be provided, *detached* from the barometers. Of the two barometers, one with its attached thermometer is to be placed in the shade, at the top of the eminence whose height is to be measured; the other remains at a station below; let them continue a sufficient space of time at each station for each detached thermometer to acquire the temperature of the air.

(2) An observer on the eminence must note down the height of the mercury in the barometer, and the temperature pointed out by both the attached and detached thermometers; *at the same time*, another observer at the station below, will make similar observations upon the instruments there placed. If, in this manner, two or three sets of observations be taken at each station at short intervals of time, the mean of their results may be taken as the true altitude.

(3) The calculation is performed as follows, viz.

Correct the height of the mercury in the barometer at the upper station, for the difference of temperature; this correction is found by multiplying the number of inches and decimals by the constant number .000101, and the product by the difference of the attached thermometers. If the temperature at the upper station is the least, the correction is additive, if greater, it is subtractive.

The difference between the logarithms of the observed height of the mercury at the lower station, and the corrected height at the upper, is the approximate elevation.

To this remains to be applied a correction for the temperature of the air; this correction is found by multiplying the approximate elevation by the constant number .000244, and the product by the difference between the mean of the two detached thermometers and 32°. If the mean is greater than 32°, this correction is to be added to the approximate elevation; if less, it is to be subtracted.

It may often happen that but one barometer can be procured, and that in consequence the observation cannot be performed at the same time at both stations. The result in this case is by no means as accurate as in the other. Were we acquainted with the effects that the presence of moisture produces on the pressure of the air, or knew the law of the decrease of temperature in rising from the surface of the earth, a single barometer, and successive observations, would suffice.

Professor Leslie has laid down a formula for estimating the decrease of temperature, but experiments are still wanting to test its accuracy. See Leslie, *Geometry.*

With regard to the effect of moisture contained in the air but little is known, for we are as yet without good instruments for even measuring its quantity.

When the process we have just described is conducted with care and attention, barometric measure is found to give results quite as accurate as the best trigonometric operations.

The remarkable coincidence of m in the barometric formula with the number 10,000, when the height is to be estimated in English fathoms, facilitates the calculation extremely.

When the altitudes of the mercury are observed in parts of the French metre, and the height is to be found in that measure, the calculation is by no means so simple, for in that case $m = 18{,}393$. To do away this difficulty, Biot has constructed tables that by a simble subtraction give the heights in metres; these tables are accompanied by others for the two corrections. See BIOT, *Astronomie Physique*, Vol. 4. MALORTI DE MARTEMONT, *Military Topography.*

In elevations not greater than a mile the following rule of Professor Leslie's will be found to give results very near the truth; as the sum of the two mercurial columns at the two different elevations is to their difference, so is the constant number 52,000 to the approximate height.

It has been proposed to substitute the altitudes of mountains measured by the barometer for horizontal bases in geodotic operations; and this method has actually been put in practice by Humboldt, in Mexico, with great success.

263. The boiling point of water, which, under the mean pressure of the atmosphere at the surface of the Earth is 212° of Fahrenheit's thermometer, diminishes with the decrease of this pressure. It has, for this reason, been proposed to substitute for barometric measurement, an observation on the temperature of boiling water.

Although the apparatus in this experiment is both cheaper and more portable than the barometer with is accessaries; the experiment itself is liable to fallacies from which that with the barometer is free.

Use of the Barometer to predict changes in the weather.

264. The height of the mercury in the barometer, even at the same place, is by no means constant, but is found to vary with heat and cold, and with the changes of the weather; the corresponding change in the barometer is known to precede those that take place in the atmosphere; hence, if carefully watched, it may enable us to foretell the phenomena of the weather.

For this purpose, Dr. Halley has proposed the following rules:

(1) In calm weather, when the air is inclined to rain, the mercury is commonly low.

(2) In serene and settled weather, and also in calm and frosty weather, the mercury is generally high.

(3) Upon very high winds, although not accompanied with rain, the mercury sinks lowest; regard being had to the point of the compass from which the wind blows.

(4) The greatest heights of the barometer, are found upon easterly and north easterly winds; to which it may be added, that under a southerly wind it is commonly low.

(The above four observations hold good in every part of Europe; but in North America they are not entirely correct; for instance, the greatest heights of the barometer in New-York take place under westerly and north west winds.)

(5) After very great storms of wind, when the mercury has been for some time low, it generally rises very fast again.

(6) The more northerly places have greater alterations of the barometer, than the more southerly.

(7) Within the tropics and near them, there is little or no variation of the mercury in all weathers. At St. Helena it is little or nothing; at Jamaica 3-10ths of an inch; whereas in England, it is 2 1-2 inches; and at Petersburgh, nearly 3 1-3d. See REES' CYCLOPÆDIA, article *Barometer.*

We shall account for these changes, and their connexion with the weather, when we come to treat of the air as *a vehicle of moisture*, and of the *winds.*

OF THE SPECIFIC GRAVITY OF ELASTIC FLUIDS.

265. As the densities of elastic fluids are small, when compared with water, experimental philosophers, in order to render the difference between them more sensible, make use of some other substance as the unit. Some have for this purpose em-

ployed the air of the atmosphere, others oxygen, and others again hydrogen.

As the last is the lightest of all known substances, it possesses advantages of a peculiar kind over the other two; the numbers too that represent the specific gravities of bodies in terms of hydrogen, as the unit, have a most remarkable coincidence with those that mark the proportions in which bodies combine chemically with each other. On the other hand, atmospheric air has this to recommend it, that it has been found to be uniform in composition in every part of the globe.

266. The comparative weight of equal volumes of atmospheric air, and of any other gas, may be found as follows:

Take a globular flask of thin glass, whose capacity is at least five or six quarts; it must not be less than this, otherwise the errors that necessarily take place in weighing, would have too great an influence on the result. This flask must be provided with a stop cock that will effectually cut off all communication with the external air. If the stop cock be opened, and the flask screwed upon the plate of an air pump, the air may be exhausted, and a partial vacuum formed within the flask. Let the stopcock be now closed, the flask detached and weighed by means of an accurate balance. Call the weight thus found P.

If the stop cock be opened, the air of the atmosphere will enter the flask, and the arm of the balance at which it hangs will preponderate; let it be again accurately weighed, and call its weight P' It is evident that the augmentation of weight is due to the air that has entered the flask; if we suppose the vacuum to have been perfect, $P'-P$ will be the weight of the bulk of atmospheric air contained in the flask.

In the same way the weight of an equal bulk of some other elastic fluid may be found. The flask is first to be weighed empty; let p be its weight; this is not essentially equal to P, for it will depend upon the density of the atmosphere at the time this last experiment is made. It is then to be filled with the gas, and again weighed; if the weight, when full of gas, be p' $p'-p$, will be the weight of the contained gas; and if s be the specific gravity in terms of atmospheric air:

$$s = \frac{p' - p}{P' - P}$$

Biot, *Physique*, Vol. I.

The same formula may be employed to find s in terms of an equal bulk of hydrogen, by substituting for P′ the weight of the flask filled with that gas.

267. If this process be repeated with the same gas, atmospheric air from the same place, the same air pump, the same flask, and the same balance, the result will be found different; this shows, that however exact the experiments may be, they cannot be compared together until they have corrected for errors arising from the following circumstances, viz.

(1) The pressure of the atmosphere is not constantly the same; the air contained in the flask, when it is weighed full of air, is acted upon by this pressure, and its mass will be of greater or less weight, according as the pressure of the atmosphere is, at the moment, a greater or less quantity.

(2) The pressure of the atmosphere is not the sole cause of a change in the weight of a given volume of air; change of temperature produces a similar effect.

(3) These two causes have a proportionate influence upon the weights of all the other gases, when they are introduced into a flask whence the air has been exhausted. The temperature and pressure at which they are introduced into the flask must therefore be noted.

(4) The flask itself does not always continue of the same capacity; for the glass of which it is made is contracted or dilated by change of temperature.

(5) Atmospheric air and the gases are capable of containing a certain quantity of moisture; this varies with the temperature, and with the care that has been taken to free them from water; the same quantity of gas weighs either more or less, as it contains more or less vapour. In order, then, to compare the results of experiments on this subject, we must know the quantity of vapour contained in the gas, as well as in the air of the atmosphere.

(6) All these causes influence the experiment in another manner, by changing the density of the exterior air in which the flask, when it is weighed, is, as it were, immersed. The loss of weight of the flask, whether empty or full, when weighed in the air, will vary, with its own bulk, with the pressure of the atmosphere, with the temperature and hygrometric state of the exterior air.

(7) We have supposed in these experiments that the vacuum produced by the air pump is perfect; this, as we have shown, is never the case; for whatever pains be taken to exhaust the flask, there must always remain in it a small quantity of elastic fluid, whose existence is shown by its action on the gauge of the pump. The quantity of this elastic residuum must be measured; it must also be ascertained whether it be entirely gaseous, or entirely aqueous vapour, or whether it is composed of a mixture of the two, and in what proportions. Biot, *Traite de Physique*, Vol. I.

OF THE ATTRACTION OF COHESION.

The laws of the equilibrium of fluids, deduced *à priori*, are in some measure influenced by a species of attraction that frequently takes place between their particles and those of solid bodies. This is called the attraction of cohesion, or from one of its most remarkable cases *Capillary Attraction*. The motion of fluids is affected even more than their equilibrium, by this cause, it is in consequence necessary to consider its phenomena before we can pass to the subject of Hydrodynamics.

268. If a polished plate of some solid substance be suspended horizontally from one of the arms of a balance, and if, after having counterpoised it by weights in the opposite scale, it is brought in contact with a liquid, it will be found to adhere to it. This is made manifest by the fact that they cannot be separated without the evident exertion of a determinate force. Biot, Vol. I.

This adhesion is not caused by the pressure of the atmosphere, for it takes place even in vacuo.

When, by the action of weights applied to the opposite arm of the balance, the plate is raised, it will be found to carry with it a portion of the liquid. It is evident, therefore, that the added weight is not the measure of the attraction between the plate and the fluid, but of the attraction existing between the particles of the fluid body itself. BIOT, Vol. I.

269. When capillary glass tubes are dipped into water, it will be found to rise in them. When two plates of glass are immersed in the same liquid, parrallel to, and at small distances from each other, a portion of the fluid will also rise between them.

When the altitude of the column of water is measured, it will be found to have risen highest in the tubes of smallest bore. This phenomenon is found to occur in a greater or less degree with all liquids that are capable of moistening glass, and although the rise is most perceptible in small tubes, it appears in some degree in all vessels whatsoever, as is shown by a ring formed around the sides with a concavity on its upper side.

Tubes of other materials than glass are capable of exerting capillary attraction on water, as are also the natural pores of many bodies, both organic and inorganic. The capillary suspension takes place, whether the tube remain immersed in the liquid or not, provided there be a drop of the liquid adhering to the lower end of it.

If the liquid and the solid body be of such a nature that they do not adhere, the liquid, instead of rising in the tube, is depressed; thus, mercury does not rise as high in a glass tube as its own original level.

270. The surface of those liquids that rise in capillary tubes, will be found on examination to be concave upwards; and in cylindrical tubes, where the liquid moves with freedom, the concavity is found to be nearly a hemisphere, touching the interior of the tube. The surface of a liquid that is suspended by capillary attraction between two parallel plates, is also concave upwards, and of the form of the half of a cylinder. The following are some of the more remarkable phenomena of capillary attraction, as observed by experiment:

(1) The surface of those fluids that are depressed in capillary tubes is convex upwards. The heights to which the same fluid rises in

cylindrical capillary tubes of different diameters, are inversely as the diameters.

(2) The height to which a fluid rises between two concentric tubes, is half that to which it rises in a tube of the same material, and whose diameter is equal to the distance between the concentric tubes.

(3) The height to which a fluid rises between two parallel glass plates is half that to which it would rise in a tube whose diameter is equal to the distance of the plates.

(4) If the plates, instead of being parallel, are inclined to each other, the surface in contact with the plates, instead of being a horizontal straight line, becomes curved, and this curve is an hyperbola.

(5) Whether a fluid rise or fall between two vertical and parallel planes, immersed in the fluid, the planes tend to approach each other. It is for this reason that two small vessels of glass, floating on mercury or water, tend to come together whenever they are placed near to each other.

271. All these circumstances prove that an attraction exists between glass and water, and between certain other solid and fluid bodies. In others, as between mercury and glass, there is an apparent repulsion. If, however, a small globule of mercury be placed upon dry paper, and then touched with a tube of glass, the mercury will adhere to the glass, and may, by means of it, be lifted from the paper.

As this experiment shows there is no actual repulsion between mercury and glass, philosophers infer that the reason mercury is depressed in capillary tubes is, that its particles have an attraction for each other, much more powerful than they have for the particles of the glass.

272. The attraction of cohesion is also observed between solid bodies; but they must be brought into close contact with each other; in other cases, they may be made to adhere closely by the intervention of a fluid.

On this last principle, we can explain the process of soldering metals.

273. The sphere of the attraction of cohesion is different in

different bodies, but is at most small; the law of its decrease in terms of the distance, has not yet been ascertained.

274. CLAIRAUT has proved, that if the law by which the particles of a tube attract a liquid be the same as that by virtue of which the particles of the fluid attract one another, the fluid will rise above the level, whenever the intensity of the first of these attractions exceeds half the intensity of the second. If it is exactly half, the surface of the interior column will be a plane, and level with the surface without. If it is less than half, the surface will be depressed and convex upwards. LAPLACE, *Mecanique Celeste,* Supplement au X Livre. CLAIRAUT, *Figure de la Terre.*

275. If thus, in consequence of the joint action of these two attractions, the surface of the fluid in the tube is rendered either concave or convex, the equilibrium between the outer and inner masses of fluid is destroyed; in the first case, the pressure of the enclosed fluid is rendered less than that of the outer mass, it will therefore rise in the tube until the weight of the column becomes equal to the difference of the two pressures; in the second, the pressure of the enclosed fluid is increased, it therefore falls in the tube.

This is the theory of LAPLACE; it is illustrated at length, and applied to the explanation of the phenomena by him, in the supplement to the 10th book of his *Mecanique Celeste.* This theory not only accounts for all the phenomena, but is very remarkable for its clearness and scientific precision.

The theory usually given in the English books is Dr. Jurin's; but it is very inferior to that of Laplace.

The analytic expression deduced by Laplace, is composed of two terms; the first of these is much greater than the second, and expresses the attraction of the mass of fluid that is terminated by a plain surface. Upon this term, in his opinion, depends the refracting power of transparent bodies, the attraction of aggregation, determining the three several states in which bodies exist, and the phenomena of chemical affinity. The second term expresses that part of the action which is due to the sphericity of the surface. On this term capillary action depends. LAPLACE, *ubi supra.*

MECHANICS.

SECTION FOURTH.—HYDRODYNAMICS.

OF THE MOTION OF FLUIDS.

The investigation of the laws that govern the motion of fluids is attended with difficulty, in consequence of our ignorance of the nature of the particles of which this species of bodies is composed, and of the manner in which they act upon each other. We are in consequence compelled to deduce the fundamental proposition of Hydrodynamics from an hypothesis.

This hypothesis considers a mass of fluid to be contained in a vessel whose sides and bottom are indefinitely thin; an orifice is supposed to be pierced in the bottom or side of this vessel, through which the fluid is propelled by the action of its gravity; we then assume that the fluid is divided into a number of horizontal *laminæ*, each of which descends perpendicularly downwards. Every particle is therefore supposed to descend in a vertical line. Upon this hypothesis,

276. The velocity with which a fluid issues from a very small orifice, pierced in the side or bottom of a vessel, is the same that a heavy body would acquire in falling through the perpendicular altitude of the surface of the fluid above the level of the orifice. Bossut, *Hydrodynamique*, § 218.

If g be $32\frac{1}{6}$ feet, h the altitude of the fluid, and v the velocity:

$$v = \sqrt{2gh}$$

De Buat, *Hydraulique*, § 3.

If the vessel be of such a figure, and the orifice so placed that the fluid may spout directly upwards, it will rise to the level of the surface of the fluid in the vessel.

If the vessel be kept constantly full, and if a be the area of the orifice, t the time of discharge, g and h as before; g, the quantity discharged, may be found by the equation

$$y = a\,t\,\sqrt{2\,gh}$$

if $t = 1$ second

$$q = a\,\sqrt{2\,gh}$$

hence,

$$a = \frac{q}{\sqrt{2\,gh}}$$

And because $v = \sqrt{2\,gh}$, the quantity of fluid discharged in a given time, is equal to the contents of a cylinder or prism, whose base is equal to the area of the orifice, and whose altitude is equal to the space that would be described in that time, with the velocity due to the height of the fluid.

If orifices be pierced in the side of the vessel at different altitudes, the velocities and quantities discharged are as the square roots of the depths of the orifices beneath the surface of the fluid.

277. If a vessel be permitted to empty itself through an orifice pierced in the side or bottom, the velocity with which the fluid is discharged is uniformly retarded. The time in which it will empty itself is double that in which an equal quantity of fluid would be discharged from an equal and similarly situated orifice in an equal and similar vessel; and the quantities discharged decrease in equal times as the series of odd numbers.

Upon this last principle the Clepsytra or water clock may be constructed; thus, if the fluid contents of a prismatic vessel that discharges itself in 12 hours, be divided into 144 equal parts; 23 of these will flow out during the first hour, 21 during the second, and so on.

278. If a fluid issue through a horizontal or oblique orifice, in the side of a vessel that is kept constantly full, it will describe a parabola in a vertical plane, passing through the axis of the orifice, and whose directrix is the intersection of this plane with the surface of the fluid produced.

This is a necessary consequence of the theory of projectiles, laid down in Section I.

279. If a vessel be filled with water, and placed upon a horizontal plane, and if, upon the altitude of the fluid as a diameter, we describe the circle, the distance to which the fluid will spout

along the horizontal plane, from a horizontal orifice pierced in the side of the vessel, is proportioned to the ordinate of the curve, drawn through that point.

If the orifice be at the centre of the circle, the horizontal range will be the greatest possible; and at equal distances above and below the centre, the ranges will be equal.

OF THE DISCHARGE OF LIQUIDS THROUGH ORIFICES AND PIPES.

When the foregoing theory is submitted to the test of experiment, it is not found to agree exactly with observation, for,

280. When a liquid descends in a vessel towards an orifice, pierced in its bottom, the thread of fluid immediately above the centre of the orifice, is the only one that descends perpendicularly. Other portions of the liquid describe curved lines, and reach the orifice in directions approaching more or less to horizontal. In the same manner there is a deviation from the right line direction when a fluid approaches an orifice in the side of a vessel.

This motion may be rendered evident, by throwing powders into the fluid whose specific gravity is a little greater than its own. BOSSUT, § 312.

281. The section of a jet that issues from an orifice, does not continue of the same magnitude with the area of the orifice, but is contracted. The actual discharge is not in consequence equal to that which is deduced from the hypothesis.

Sir Isaac Newton first discovered a contraction in the jet, and thought it to be in the proportion of 5 to 7, or as the square roots of 1 and 2. If it were of this amount, the quantity discharged might be represented by $a \sqrt{gh}$. From this, it was erroneously concluded, that the velocity was due not to the height of the fluid, but only to its half. This is not the case, for the decrease is in the area of the vein, and not in its velocity.

282. The narrowest part of the jet is called the *Vena Contracta*; its distance from the orifice, as determined by the ex-

periments of Bossut, is equal to the radius of the orifice, and its area is to that of the orifice, as 62 to 100, or as 5 to 8. BOSSUT, § 359.

283. Beyond the vena contracta, the fluid again spreads in the form of a frustum of a cone, the angle of whose vertex is 32°.

284. From the researches of Bossut, the following deductions are obtained, with regard to the discharge of fluids, from vessels kept constantly full.

(1) The actual discharges are less than the theoretic in the proportion of 62 to 100, but are *nearly* proportioned to the square roots of the depths of the fluid. BOSSUT, § 355.

(2) If the altitude of the fluid be constant, but the area of the orifice variable, the discharges are *nearly* proportioned to the areas of the orifices. BOSSUT, § 354.

(3) In general, the quantities discharged, during a given time, through different orifices, and at different depths, are *nearly* in the compound ratio of the areas of the orifices and the square roots of the depths. BOSSUT, § 356.

(4) These laws are not exactly, but only *nearly* true, in consequence of friction. To this cause it is owing, that small apertures discharge rather less than the calculated quantity; and that in orifices of equal areas, but different figures, those with the smallest circumferences will, under the same circumstances, discharge the greatest quantity of fluid.

285. If the fluid, instead of passing through an aperture pierced in a thin plate, be made to pass through a tube or *adjutage*, the discharge will be much increased. If the tube be cylindrical, and of the same diameter with the orifice, the hypothetical discharge, the discharge through an additional tube, and the discharge through a thin plate, are to one another as the numbers 16, 13 and 10. BOSSUT, § 384.

Other adjutages are attended with different degrees of advantage. A full set of experiments has been made upon them, by Professor Venturi. See CAVALLO's *Philosophy*, Vol. II. chap. 7.

From them it appears, that where water is to be admitted from a reservoir into a pipe, through which it is to discharge itself, the part of the pipe adjacent to the reservoir should be made of the form of the *vena contracta;* while the opposite extremity of the pipe should be also conical, its sides diverge from the axis at angles of 16°.

286. When a prismatic, or cylindrical vessel, is permitted to empty itself through an orifice ; if the quantity that would be discharged in a given time be calculated upon the hypothesis in section 276, and reduced in the proportion of 5 to 8 ; it will afford results that will be accurate, until the fluid shall have descended nearly to the bottom ; at this time a funnel shaped cavity is formed at the surface, and a whirling motion takes place, that renders the discrepancy still greater ; for this no adequate correction has yet been found. BOSSUT, § 433.

On this account, in the clepsydra, no dependence can be placed on the regularity of the discharge of the last portion of fluid.

OF SPOUTING FLUIDS.

287. A jet of fluid whose direction is perpendicularly upwards, should, according to theory, rise to the same height with the surface of the mass contained in the reservoir. This is far from being the case in practice; for the rising fluid is subject to the action of several retarding forces.

The most obvious of these are the friction on the sides of the orifice, and the resistance of the air. But the particles of the fluid have, in addition, to push before them the particles that have preceded them whose velocity is diminished, and to overcome the resistance of those that after having risen to the height of the jet, and lost their whole initial velocity, are again descending.

It is evident that there must be a limit both to the diameter and the altitude of the jet. Cavallo states the former to be 1¼ inches, the latter 100 feet.

288. If we know the diameter of a pipe drawn from a reservoir that just supplies the expense of a jet of given diameter, we may calculate the diameter of a pipe that will supply a jet of any other diameter, for the squares of the diameters of the pipes that convey the fluid should be to each other in the compound ratio of the squares of the diameters of the orifices, and the square roots of the heights of the reservoir.

289. Oblique jets are likewise affected by the resistance of the air, and friction on the sides of the orifice. Where, however, the velocities are small, the curve they describe does not differ sensibly from a parabola, but it is of less magnitude than that pointed out by the theory, and its directrix is at the altitude of the effective head. Bossut, § 486.

Jets of water in various directions, known by the name of *Fountains*, were formerly much used in the embellishment of gardens. Their theory was, in consequence, considered as of considerable importance, and occupied a large space in books upon hydrodynamics. They have, however, of late years, fallen almost altogether into disuse.

OF THE MOTION OF WATER IN PIPES AND IN OPEN CHANNELS.

The theory of the motion of water in channels, whether open or closed, has been reduced to fixed principles by Du Buat, and is given by him in his *Principes d'Hydraulique.* Other researches have been made more lately, by several ingenious persons, but nothing of real importance appears to have been added to the formula and facts stated by Du Buat. His fundamental proposition, and which is equally applicable to open or close channels, is as follows, viz.

290. When water flows uniformly, in any bed whatever, the accelerating force that causes its motion, is equal to the sum of all the forces that retard its progress. Du Buat, *Discours Preliminaire.*

Water, in flowing in a channel, is retarded by its friction against the sides and by its own viscidity.

291. The moving force of each of the particles of water, that compose a river, is due to the slope of its surface, and to no other cause. The motion, too, of water in a tube, whether horizontal or inclined in any manner whatsoever, may be referred to its motion in an inclined tube of uniform descent, loaded with a head of water of such a height as would produce the motion in the tube. For this reason, a pipe may be considered as a river of uniform slope, and whose bed is constant. Du Buat, § 15, § 20.

In the calculations and formula of Du Buat, he makes use of an inch as the unit of length, and expresses the slope by the fraction $\frac{1}{b}$; where b is the distance in which a change of level equal to 1 inch takes place. Bu Buat, § 15.

292. The friction of water against its bed appears to differ from that of solid bodies against each other, in being independent of pressure, and having relation only to the surfaces and velocities. Du Buat found it to vary with the square root of the mean depth, and in a ratio less than the square of the velocity.

The Hydraulic mean depth is a line that, multiplied by the length of the section of the bed, gives a surface equal to that of the section of the stream. In circular channels, it is equal to half the radius.

293. The mean velocity per second of a fluid running in a channel with equable motion, may be represented by the following formula:

Where V is the mean velocity in English inches,
r the mean depth,
b the denominator of the fraction
$\frac{1}{b}$ representing the slope,
and L the hyperbolic logarithm.

$$V = \frac{307.\ \sqrt{r - 0.1}}{\sqrt{b}\,\mathrm{L}.\ \sqrt{b - 1.6}} - 0.3\ \sqrt{r - 0.1}$$

294. A current is in train, where the fluid runs in it uniformly, forming neither waves upon its surface, nor eddies within it. Each separate thread of fluid has then its own proper velocity

that is constant. The mean velocity of the current is that which, multiplied by each point in the section of the bed, gives a product that is equal to the sum of the products of each point, in the section, into its own velocity. DU BUAT, § 64.

295. The relation between V the mean velocity, v the velocity at the surface, and U the velocity at bottom, may be expressed by the following equations, viz.

$$v = (\sqrt{V - 0.25} + 0.5)^2 = (\sqrt{U} + 1)^2$$

$$V = (\sqrt{v} - 05)^2 + 0.25 = (\sqrt{U} + .5)^2 + 0.25$$

$$U = (\sqrt{V} - 1)^2 = (\sqrt{V - 0.25} - 0.5)^2$$

In order to facilitate the calculation, by means of these formula, Du Buat has constructed a table, by means of which v and U may be found when V is known. See DU BUAT, § 65.

The velocity at the surface of a stream, in an open channel, may be found by means of an instrument invented by *Pitot*, and called after his name, *Pitot's* tube; for a description, See DU BUAT, § 572, and BOSSUT, § 657.

Another and better instrument has been contrived by Gauthey. See his *Memoires*, Vol. III.

296. When the channel in which a fluid runs is not straight, the resistance to its motion is increased; when the channel is composed of a number of straight parts joined by elbows, the increase of the resistance may be calculated on the following principles.

(1) The resistance caused by elbows in the channel, increases with the square of the velocity of the fluid.

(2) It is proportioned to the square of the sine of the complement of the angle, that the prolongations of the parts of the channel make with each other, provided it do not exceed 36°; beyond this limit, the resistance increases more rapidly.

(3) In the same channel, the resistance increases with the number of elbows or bends in its course. DU BUAT, § 104.

Of Rivers.

297. Rivers are natural channels, through which the water proceeding from springs, or from the rain that falls and runs over the surface of the ground, is conveyed to the sea.

The beds of rivers appear to have been formed by natural causes, operating for a long series of years, and we generally find them in a permanent state, subject neither to increase or diminution, except within limits that do not affect the magnitude of their channels, or the stability of their banks. The tendency of nature, to produce an equilibrium among active and passive forces, is no where more strongly marked, than in the case of the beds of rivers; for we find that a river always runs over ground exactly suited to the velocity of its current. Thus, where it is very rapid, the bottom is solid rock, as the rapidity diminishes, we find large rolled stones, then gravel, sand, and lastly mud and clay. The water, by its own action, carries off the finer particles from those places where it is most rapid, to deposit them where it is less so.

It sometimes happens, however, that a river is not in train, or that it is subject to a periodical increase of its waters; and owing to either of these causes, it may be a source of ruin and devastation to its borders, instead of being their ornament, the vehicle of their commerce, and the cause of their fertility. In such cases, we must call in the aid of art, and art can only be successful, by copying the means that nature has herself made use of, in similar cases: For instance,

298. When a river that occasionally overflows its banks has many sinuosities, as we know that the velocity and consequent discharge will be increased with the quantitity $\frac{1}{l}$; we may cut through a number of the points, and thus, by shortening the course, render the rapidity and consequent discharge greater.

299. When this will not suffice, or is inapplicable, we erect dikes, thus imitating the course of nature, in which we always find the banks of those rivers that are subject to inundations higher than the surrounding country.

300. One of the most important purposes to which rivers are subservient, is that of navigation. Some rivers are to be found fitted for this purpose without any aid from art; but there are also many, where the intervention of human ingenuity is necessary to overcome certain obstacles, or obviate difficulties that naturally exist.

(1) Streams running in mountainous countries, frequently pass abruptly from one level to another, forming *Falls*.

(2) Their beds are often traversed by rocks, or other strata, too hard to be removed by the action of their current; in this case, an accumulation of water takes place above the obstacle, until the quantity $\frac{1}{b}$ becomes sufficiently great to cause a change in the velocity sufficient to discharge the whole stream. Such obstacles are called *Rapids*.

(3) Rivers, in descending from a mountainous country, carry with them large quantities of sand, gravel, &c.; when they reach the plains, this is deposited; the stream spreads itself over a great extent of ground, and becomes too shallow for navigation.

(4) Where a river meets the waters of the ocean, it is affected by their tides, and at the alternation of ebb and flow, it becomes for a short space of time absolutely quiescent. At this moment it will deposit any remains of earthy matter it may carry with it, and thus form a bar at the mouth, or even islands. In process of time these islands increase in magnitude, and finally join the main land. In this way the deltas of rivers are formed, of which we have instances in the Nile, the Mississippi, &c. The latter river, in the short space of time that has elapsed since its first discovery, has extended a deposit 50 miles into the Gulf of Mexico.

(5) A river may be unobstructed, and deep enough for the purposes of navigation, but so rapid as not to admit of an ascending trade.

301. Each of these obstacles admits of an appropriate remedy, except in cases where its magnitude is beyond the reach of human exertion, or where the cost would exceed the value of the benefit.

(1) Where the navigation of a river is interrupted by a fall, we may make a lateral channel or *canal*, and apply locks or sluices.

(2) The same remedy may be applied to the case of rapids.

(3) As shallows arise from a diminution of the velocity of the stream, they are to be remedied by making it greater; this is to be done by contracting the dimensions of the channel. Piers may be built at intervals, and alternately from each bank; the whole deposit will take place between the pier heads, and the stream will not only maintain its former depth, but will clear itself a deeper channel.

(4) When a river is of moderate size, bars at the mouth may be removed upon a similar principle: two piers are to be built into the sea from the two shores, inclining to each other at a small angle, until they approach near enough to give the river a velocity in flowing through them that will overcome the action of the tide.

(5) When a river is too rapid for the ascending trade, dams may be built across it at intervals; the whole of the fall will thus be transferred to them; in each of these dams a gate or sluice is made; when vessels are to descend this is opened, and they pass with great velocity; when the water has run out so far that the upper and lower ponds are nearly of the same level, the ascending trade may be drawn by main force through the sluice.

This contrivance, however simple and rude it may appear, was unknown to the ancients.

In most cases, however, a double sluice, or gates enclosing a chamber called a lock, has now been substituted. It has very great advantages, even in rivers, over the old sluice; and without it the canals of modern times, that frequently pass through great differences of level, would have been impracticable. Sluices of the original form are used in China, where locks are unknown, and may still be seen in Italy, in Flanders, and in Holland, where there is no great change between the levels of continuous navigable waters, whether natural or artificial.

Of Canals.

302. Navigable Canals are sometimes made use of, as has been already stated, to form a communication between different levels of a stream, whose navigation is interrupted by falls; at other times, they are formed along the bank of rapid rivers, as the least expensive mode of improving their navigation; at others they receive the waters of a navigable stream, and convey its trade to one whose level is lower; or, finally, they may

be made channels of communication between navigations that are lower than the canals themselves.

Canals were constructed by several of the ancient nations; but as they were ignorant of the use of sluices and locks, they were confined to plain countries, and were carried along the same level for their whole course. Those which are carried over levels higher than either of the streams with which they communicate, and that receive their waters from feeders and reservoirs, at their summit level, are justly considered among the noblest triumphs of human genius. Sluices were invented in the 11th century, and the first are supposed to have been erected on the canal of Martizana in Italy.

303. A lock is a chamber, interposed between two contiguous levels of a natural or artificial navigation; it is enclosed on the lower side by a gate, and its bottom is on a level with the bottom of the lower part of the canal; on the upper side, it is enclosed by a wall rising to the height of the bottom of the upper level, and this is surmounted by a gate. The opening or shutting of these two gates, establishes or cuts off the communication with either level at will. When a boat is to rise from the lower level, the lock, if full, is emptied; the boat is then introduced into the basin, and the lower gate closed behind it; water is now admitted through channels contrived for the purpose in the walls, or by wickets in the upper gate; when it shall have risen to a level with the upper portion of the navigation, the upper gate is opened, and the boat is drawn forwards.

The size of a lock will depend upon the magnitude of the navigation, and the size of the vessels that are used upon it. Where the boats are not intended for any other purpose than the navigation of the canal itself, those which are drawn by a single horse are the most economical; a lock for the passage of these need not exceed 60 feet in length, and 5 in breadth. The locks on the great canal in the state of New-York, are 90 feet long, and 16 feet wide; the canal itself is 4 feet deep.

Canals should be at least a foot deeper than the greatest draught of water of the vessels intended to navigate them. This will give room for partial deposits of mud, allow for evaporation and waste, that it may not be possible to supply instantaneously; and what, in a well planned canal, is of more importance, will do away a resistance analogous to friction, that becomes very visible when boats nearly fill up the channel in which they move.

In locks of the above dimensions, the upper gate may be made in a single leaf, but in larger locks, the upper gates are made of two parts, and so are the lower gates in all cases. By making the gates to fold, the two parts meeting at an angle, whose vertex is opposed to the accumulation of fluid, great additional strength is obtained.

There is an analytic investigation of the proper angle, at which the gates of a sluice should meet each other, to be found in the second Vol. of the American edition of Hutton's Course. It does not, however, appear to carry conviction with it. The best engineers make the vertex of the angle to project towards the head of water, from $\frac{1}{3}$ to $\frac{1}{4}$ of the whole breadth of the lock. GAUTHEY, *Memoires*, Vol. III.

The fall of a single lock should not exceed 10 feet; where the change of level is greater than this, it should be divided among two or more locks. The limit to the height of a lock, is found in the additional expense of construction in masonry and wood, and the greater risk of destruction in deep, than in shallow locks; the thickness of a wall that will sustain the pressure of a given head of water, is equal to half the depth of the water. GAUTHEY, Vol. III.

For the dimensions of the timber of the gates, in order to resist given pressures, See GAUTHEY, Vol. III, p. 75, et seq.

Deep locks also cause a much greater expenditure of water, than where the whole fall is divided among several; for this reason, where circumstances have compelled engineers to make use of a single lock, they have endeavoured to save the water by several ingenious contrivances. For a lock of this description, See BELIDOR, Vol. IV.

Wherever it is possible, a good engineer will endeavour to make all his locks of nearly equal fall. Or a better rule is, that they shall diminish in depth in a small degree, as they recede from the point where the supply of water is admitted. Where an additional supply of water is brought in at a lower level, the locks may again be constructed of the maximum depth. By an attention to this rule, the water used in the first lock will carry a boat from the highest to the lowest level, without requiring any increase, or lessening the general depth of the canal. GAUTHEY, Vol. III.

Locks of the fall of 8 feet, should never be placed nearer to each other than from 1 to 200 yards, (according to their superficial dimensions,) unless the width of the intermediate canal be proportionably increased. Where they are nearer than this to one another, they cause a very great waste of water.

Where locks are 200 yards apart, and the boats pass upwards and downwards alternately, each boat will require no more than a single lock-full of water; where they are at the same distance, and the boats follow each other, the expenditure will be equal to two locks for each boat; the nearer the locks are brought to each other, the greater will be the expenditure, until when they are adjacent, and the lower gate of one lock forms the upper gate of the next, the expenditure in boats passing alternately upwards and downwards, will be equal for two boats to the fall of as many locks as are combined in the system. GAUTHEY, Vol. III.

A want of attention to this last circumstance, has led many engineers into great errors. For it has been usual to combine as many locks together, at one place, as possible, and yet, in calculation, no more than two locks of water have been allowed for each boat.

304. One of the chief difficulties which attends the construction of a canal, is that of procuring an ample supply of water for the purpose of lockage, and to meet the waste occasioned by leakage, and by evaporation from the surface.

The quantity of evaporation is about 3 feet per annum; the leakage will depend upon the nature of the banks, and the lockage on the number of boats that ascend and descend, and a more or less favourable construction and location of the locks. It is usual in Canal estimates to allow twice as much water for leakage as for evaporation.

305. Where the canal, from its superior elevation, or from any other cause, cannot be furnished with water from either of the navigations in which it terminates; *Feeders* must be drawn from some source, either natural or artificial.

The quantity of water that a given feeder will furnish, or the magnitude and slope of a feeder, that will furnish a required quantity of water, may be calculated by an application of the formula of Du Buat, given at § 293.

306. Where there are no sufficient natural sources of supply, or where they are subject to failure at certain seasons of the year, ***Reservoirs*** are constructed; in these the waters of springs, small streams, &c. that would otherwise run to waste, are collected in the first case; and in the second the excessive supply of some seasons is treasured up against a time of drought.

Reservoirs are also of use to receive the water of rivers, &c. before it is delivered into a canal, and retain it until it shall have deposited the mud and sediment which they have carried with them. It is extremely injurious to a canal to permit water to be introduced into it, until it shall have undergone this process of purification.

307. For the same reason, where a stream intersects the course of a canal, and their several waters are nearly at the same level; instead of admitting it into the canal, it is often necessary to carry it beneath its bed; this is done by means of a contrivance called a culvert. BELIDOR, Vol. IV.

308. When the level of the intersecting stream is higher than that of the canal, it may be carried over it, by means of an aqueduct bridge; in like manner, if the level of the canal be the highest, a similar bridge may be used to support its waters.

Some of these aqueduct bridges cross streams of considerable size. In the case of the Duke of Bridgewater's canal, that crosses the Irwell, near Manchester, vessels may be seen navigating the river and the aqueduct at the same moment; one passing directly over the course of the other.

Cast iron has, of late years, been used in Great-Britain, for the construction of aqueducts, and with much advantage. The most magnificent work of this sort is on the Ellesmere canal, in the vale of Llangollen, in Wales.

It has also been proposed to use the same material in locks.

309. Where feeders, or adventitious supplies, bring into a canal more water than can be used, *Waste-gates* must be formed at intervals along it, and in places whence the superfluous liquid may be readily carried off, either by natural or artificial

channels. A waste-gate is a wedge-shaped mound, whose edge is at the level, where the water in the canal should stand.

310. There are countries where water is not to be found in quantities sufficient for the supply of the locks of a canal. Various contrivances have been proposed to meet this difficulty; among them, may be mentioned the hydraulic locks of Betancourt; the inclined plane, and the perpendicular lift. See LESAGE, *Recueil des Memoires.* REES' *Cyclopædia.* FULTON *on Inland Navigation.* REPERTORY OF ARTS, Vol. I and II.

Of the Motion of Water in Pipes.

311. Where a pipe is adapted to a reservoir of water, the fluid will fill and flow through it, whatever be its figure or the number of its sinuosities, provided no part of the pipe is higher than the level of the water in the reservoir.

312. Where the water from a reservoir is conveyed in long horizontal pipes of the same aperture, the discharges, in equal times, are nearly in the inverse ratio of the square roots of the lengths. PLAYFAIR, § 285.

313. The principle of Du Buat, that we have seen applied to rivers and canals, also holds good with regard to conduit pipes, viz. The sum of all the retarding forces is equal to the accelerating force. By the application of this principle, we obtain a solution of the two chief problems of the motion of water.

(1) To determine the discharge from a conduit pipe, where the height of the reservoir and the diameter of the pipe are given; we have, as before:

$$V = \frac{307\,(\sqrt{r} - 0.1)}{\sqrt{b} - L\,\sqrt{b + 1.6}} - 0.3\,(\sqrt{r} - 0.1)$$

and V the mean velocity being given, the quantity is found by multiplying it by the area of the tube.

In this equation, the value of b is determined by dividing the length of the pipe by h, a fictitious height of the reservoir a little less than the true. If the mean velocity v, calculated by this hypothesis, is such that

$$h + \frac{v^2}{2g} = \text{H the true height,}$$

we shall have calculated the true mean velocity; but if this is not the case, we assume another value for h, until, by repeated trials, we shall find a number that will answer the purpose. Du Buat, § 228.

If the pipe has a number of sinuosities, the angles of which are known, the resistance that each offers may be found by the formula

$$\frac{V^2 \sin.^2 o.}{3000}$$

where o is the supplement of the angle formed by the parts of the pipe at the bend. Du Buat, § 229.

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(2) To determine the diameter of a pipe that will discharge a given quantity of water, where its length and the height of the reservoir are known.

If r be the mean depth, equal to half the radius, the given quantity, p the ratio of the circumference of a circle to its diameter:

$$r = \left(\frac{2(\sqrt{b} - \text{L}\sqrt{b + 1.6})}{4p\sqrt{307}}\right)^{\frac{2}{5}}$$

Du Buat, § 230.

The motion of water in pipes has of late attracted much attention, in consequence of its great importance to the health and comfort of large cities. It is well known that the regular sources of the springs and wells that exist in cities are either entirely cut of, or contaminated by such impurities as render them offensive and unwholesome. No other resource remains but to draw supplies of water from a distance. The ancients effected this by means of open aqueducts; the moderns, where the distance is not great, make use of pipes; where the distance is great, the main stream is brought in an aqueduct, and the distribution effected by pipes.

314. Conduit pipes are liable to be obstructed by lodgments of air in the upper sinuosities of the tubes, and by deposits of sediment at the lower ones. The first of these is prevented by a valve that floats on water, and is thus kept closed while the pipe is filled with that fluid, but falls and opens as soon as air lodges, permitting it to escape, followed by the water that raises and closes the valve. Deposits of sediment in the pipe itself are prevented by making its lower part communicate with an air tight box; this receives the sediment, and when full it is removed and emptied.

OF THE PERCUSSION AND RESISTANCE OF FLUIDS.

315. The several circumstances of the percussion of fluids against planes may be represented by the following formulæ:

$$f = m.\ s.\ a^2\ v^2 \sin^2 i \qquad \text{or}$$

$$f = 2\, m\, s\, a\, ^2 g h \sin\, ^2 i$$

Where f is the force with which the fluid strikes, m a constant co-efficient, determined by experiment in the case of water, s the specific gravity of the fluid, v its velocity, h the height due to that velocity, g the measure of the force of gravity, and i the angle of the inclination of the plane to the course of the fluid. Bossut, § 719, et seq.

Where the extent of the plane is indefinite $m = 1$; where the extent of the plane is not greater than that of the section, the stream $m = \frac{1}{2}$.

In introducing s into the formula, regard is had merely to the inertia of the particles; there is, however, a cause of error arising from the different cohesive force of the particles of different fluids; but it is only perceived in slow motions, and is very small, except where the fluid under consideration has much viscidity. Playfair, § 293.

Experiment has shown that the force does not vary in the ratio of $\sin^2 i$, except where the angle i is between 60° and 90°. Bossut found that where a prow, of the form of a wedge, is drawn through a fluid, if the resistance to its base, = 10,000, the resistance to the wedge itself will be

$$10.000 \cos.^2 i + 3.153 \left(\frac{i}{60}\right)^{3\frac{1}{4}}$$

PLAYFAIR, § 294.

316. If the plane struck by the stream, be itself in motion, the impulse is as the square of the difference of their velocities. And

If the fluid be at rest, and the plane alone in motion, the resistance is as the square of the velocity of the plane.

Hence it would appear, that the resistance of a fluid to a body in motion, is the same with the percussion of a fluid moving with the same velocity against a body at rest; but although this is founded on reasoning very evident, it does not agree with the fact, for the resistance is less than the percussion in the proportion of 5 to 6. Playfair supposes this difference to arise from the action of the fluid on the hinder part of the body moving through it, by which the resistance is in some measure counteracted. It is, however, the absolute quantity, and not the ratios of the resistances that are affected, for they are still among one another, as the squares of the velocities. PLAYFAIR, § 293.

317. Supposing the resistance to oblique surfaces to vary with the square of the sum of the angle of inclination; the resistance to a cylinder, placed vertically in a stream, is two thirds of that which would be encountered by the plane, which is its section; and the resistance to a hemisphere, is equal to one half of that opposed to a circular plane of the same diameter. BOSSUT, § 734, § 735.

OF THE MOTION OF WAVES.

318. Where pressure is applied to any portion of the surface of a liquid, the column pressed upon is shortened, and sinks below the natural level of the surface; while the contiguous columns rise above that level. As soon as the last have attained a certain height, they fall and sink below the natural level of the surface, raising not only the first column, but communicating a

OF THE MOTION OF ELASTIC FLUIDS.

The general hypothesis, by means of which the theory of the motion of fluids is investigated, is applicable to elastic as well as non-elastic fluids. Of elastic fluids, that whose motion is most important, is the air of the atmosphere.

324. Air will rush into a vacuum at the surface of the earth, with the velocity a heavy body would acquire in falling through the height of a homogeneous atmosphere.

If V be this velocity, H the height of a homogeneous atmosphere,

$$V = 8 . \sqrt{H}$$

$$\text{if } H = 26000 \text{ feet}$$

$$\therefore \quad V = 1328 \text{ feet.}$$

325. The velocity with which air rushes from a vessel into one exhausted of air, is uniformly retarded, and becomes 0, when the air in the two vessels becomes of equal density.

OF THE WINDS.

326. Where the air of the atmosphere is in motion, its currents are called *Winds.* The principal cause of the winds, is the disturbance of the equilibrium of the atmosphere by the unequal distribution of heat.

This unequal distribution of heat, is partly due to the figure of the earth and its circumambient atmosphere, partly to the alternation of seasons caused by the obliquity of the equator, and partly to local causes.

(1) The intensity of the sun's rays is as the quantity that falls on any given space; hence, if a small portion of the earth's surface be considered as a plane, the intensity of the sun's rays will be as the sine of the sun's altitude above the horizon. Those places where the

sun is vertical, are of course most heated. The atmosphere being a spherical shell concentric with the earth, the rays of the sun must pass through it for a greater distance when they fall obliquely, and a greater quantity must be lost and dispersed by refraction and reflection.

(2) The meridian altitude of the sun above the horizon, is not the same two days successively at the same place; the difference in the heat received from the sun at any given place, on different days, is therefore great. According to Prevost, the proportion between the quantities of heat that reach the surface of the earth, at Geneva, in July and December, are as 7 or 8 to 1. YOUNG, § 56.

The earth does not appear to grow either warmer or cooler; the whole of the heat, therefore, which is received by the earth, radiates from it again. In summer, any given portion of the earth receives more heat than it radiates, and hence there is an accumulation of heat after the solstice; this does not reach its maximum until about a month later. The maximum of cold is, for the converse reason, a month later than the winter solstice.

(3) The temperature of any particular place, and even of whole countries, is affected by the vicinity of seas, great lakes, mountains, morasses, forests, and by the greater or less degree of cultivation it exhibits.

Seas and great lakes are more slowly heated than the surface of the earth; for the rays of the sun are not absorbed by the upper strata, but pass through, and are not entirely lost until they reach a great depth; the heat derived from the sun is therefore distributed throughout the whole mass. They are likewise more slowly cooled by radiation; for when the upper stratum has given out its heat by radiation, it becomes heavier than the one immediately beneath, and descends to give it the place due to its difference of specific gravity.

As we rise above the level of the sea, the heat is found to decrease at a rate that is stated by Playfair, as 1° of Fahrenheit for 270 feet. On ascending into the atmosphere, therefore, at any latitude, to a certain height, we may reach a point where the mean height of the thermometer is 32°. This altitude is called the height of perpetual congelation. It is elevated 15,000 feet above the level of the sea, at the equator; and at the latitude of 80° no more than 1200 feet. The cause of this diminution of heat, appears to be owing to the diminution in the density of the atmosphere, and the superior capacity of rare air for heat.

High mountains, therefore, affect the climate, and may not only temper a warm one, but increase the rigour of a cold one. We find in the torrid zone, mountains covered with perpetual snow, and plains affected with no greater heat than the most temperate climates.

Forests tend to produce cold, by preventing the sun's rays from reaching the ground. Morasses and small lakes increase the cold of a climate, by the absorption of heat the evaporation from their surfaces causes.

Cultivation affects the mean temperature, by exposing the surface of the earth to the rays of the sun. It produces an effect not only on the country where it takes place, but upon those adjacent; thus, the climate of Italy has been influenced by the destruction of the forests of ancient Gaul and Germany.

The mean temperature of some places is affected by another cause. When they are near the line of perpetual congelation, and foreign masses of ice are introduced by any accidental circumstance, they will gradually become colder; for in the alternations of the one side of the freezing point, heat is suddenly let loose, and radiates to a distance, while thaw is a slow and gradual process; the adventitious ice will therefore increase rather than diminish. We may in this way account for the accumulation of ice on the shores of Greenland, the consequent change in the climate of Iceland, and the advance of the glaciers on the mountains of Switzerland. See PLAYFAIR, § 387, et seq. LESLIE, *Elements of Geometry.*

The northern hemisphere is found, by observation, to be warmer than the southern; this is supposed to arise from the greater length of the summer on the northern side of the equator; the interval between the vernal and autumnal equinox of the same year, being 7 days longer than the interval between the autumnal and the vernal of the succeeding year.

327. The several winds whose course may be explained upon the foregoing principles, may be divided into the following classes:

1. The trade winds.
2. Monsoons.
3. Local modifications of these two.
4. The westerly winds that are found to prevail near the northern and southern limits of the trade winds.
5. The variable winds of the temperate and frigid zones.
6. Land and sea breezes.

The air over that zone of the earth which is most heated by the solar rays, is expanded, becomes lighter in consequence, and rises; the adjacent columns, to the north and south, rush in to supply the place, and thus a current is produced, that, in the absence of any other cause, would be from the north, on the north side of the heated zone, and from the south, on its south side. But in consequence of the rotation of the earth upon its axis, another motion is combined with that just mentioned; for the air acquires a velocity corresponding to that of the portion of the earth immediately beneath it, and as it comes from a latitude where the velocity is less, to one where it is greater, it will have an apparent motion in a direction contrary to the diurnal revolution of the earth, or from east to west; this motion, combined with the other, will make the direction of the trade wind at its northern limit, N. E. and at its southern limit, S. E. As it approaches towards the zone that is most heated, the velocity from N. to S. and from S. to N. is destroyed by the counteracting effect of the current flowing in from the opposite direction, and the apparent course of the wind becomes due east.

This is the cause of the trade wind, which, with certain exceptions, blows continually between the tropics, and even reaches the parallel of 30° on both sides of the equator. In summer it is to be found on the coast of America as far north as 32°.

The zone where the trade wind is due east, varies but little, and is at a mean on the north side of the equator; this is owing to the greater heat of the northern hemisphere. This theory of the trade winds, that supposes them to be caused by the air in its motion southward, falling back towards the west, is mentioned by Halley, but rejected by him; it has since been espoused by Franklin, Laplace, and Playfair, and is fully entitled to be received as true; for it not only explains the phenomena, but is derived from causes that are known to exist in nature. PLAYFAIR, § 410. FRANKLIN, *Works*, Vol. III. LAPLACE, *Mechanique Celeste.*

328. When the air has risen over the central zone of the trade winds, to a certain height, it there forms a counter current, that carries it back from the equator, towards the poles. The general direction of this upper current must incline to the westward. The zone of the trade winds being limited by the parallels of 30° of north and south latitude, the counter current descends to the earth there; hence we find, that in the north and south Pacific, and in the Indian Oceans, westerly winds are almost constant between the parallels of 30° and 40°.

Not only the trade, but all other winds, are accompanied by counter currents in the higher regions of the atmosphere; and the quantity of air in those opposite currents is so nearly equal, that the average weight of the air, as measured by the barometer, is nearly the same at all places on the earth's surface. PLAYFAIR, § 410.

The surface that would separate two opposite currents of equal pressure, is elevated about 18,000 feet above the level of the earth. PLAYFAIR, § 412.

329. The Indian Ocean is bounded by land on the north. When the sun is on the north side of the equator, this land is more heated than the water, the air adjacent to it rises, and that from the ocean rushes in to supply its place; but as it comes from a portion of the earth, whose velocity of rotation is greater than that it passes over, its apparent direction becomes S. W. When the sun returns to the south side of the equator, the land cools, and the regular trade wind from the N. E. again prevails.

These periodical winds, that are met with in the Indian Ocean, are called Monsoons.

330. Local circumferences produce many remarkable modifications of the trade winds, and monsoons. Thus, in the gulf of Guinea, the wind blows from the south and south west continually. This is explained by supposing that the S. E. trade wind, after passing the equator, inclines towards the shore, in consequence of that being much heated by the rays of a tropical sun.

Adjacent to the region of this westerly wind, is a tract of sea where perpetual calms appear to exist, attended with thunder, and almost continual rain. In this tract, the counter current of the wind, just spoken of, appears to descend again to the earth's surface.

331. The remaining portion of the earth's surface, is the seat of variable winds. In it, the changes of temperature are more rapid, and subject to less certain laws; hence, the currents are less constant in their direction, and of no definite duration. Yet, notwithstanding these irregularities, there is a tendency in most countries to periodical winds.

In Europe, a north east wind prevails in summer, to a very great degree; this is supposed to arise from the same cause as the trade winds. The Elesian wind of the ancients is this, modified by the action of the adjacent countries.

In the United States, a north east wind is likewise frequently felt; it also arises from the same great cause, and becoming yearly more frequent, during the summer months, promises to be the prevailing wind of that season.

A remarkable current, called the Gulf Stream, is to be found in the Atlantic Ocean, near the coast of the United States; a constant trade wind blowing into the Gulf of Mexico, raises the water therein above the general level, and it discharges itself with great rapidity between Cuba and Cape Florida; although it spreads, and its velocity diminishes, it continues to be felt as far as the banks of Newfoundland to the north. This current, drawing its supplies from tropical regions, is more heated than the adjacent ocean; and in winter is considerably warmer than the land of the United States; for this reason, a wind is frequently found blowing towards the Gulf Stream over the continent of North America. The direction of this is N. W. and it is the prevailing winter wind of our climate.

A continuous ridge of lofty mountains extends the whole length of both Americas; within the zone of the trade winds, the plain of Mexico is 8,000 feet above the level of the sea, and the peaks of the ridge are more than double that height; the trade wind, therefore, has not a continuous current around the earth at this place, but would pass along the sides of these mountains to the north and south, were it not impeded in the former of these directions by currents from the N. W. and N. E. as we have just explained; the accumulation must at times become very great, and cause a current. We may thus explain the S. W. wind that is often felt in the United States, particularly in the summer season.

These three winds are the great prevailing winds of North America. In New-York, out of 36 days, the wind blows 12 from the N. W. 6 or 8 from the N. E. 12 from the S. W. and the remaining 4 from all other points of the compass.

332. In countries situated within the tropics, and in the summer of other warm climates, where the sea is not far distant, a current of air is found to set daily from the sea towards the land, while at night the direction is reversed. These currents are called the land and sea breezes; during the day, the land is more rapidly heated than the sea, the air over it rises, and that over the sea presses in; at night again, the land parts rapidly with its heat, while the sea retains its temperature almost unchanged, and thus a contrary effect is produced.

These sea breezes are very frequent, even in the latitude of New-York, during the months of June and July.

333. The changes of temperature induced by the precipitation and solution of moisture, and the action of electrical phenomena, will suffice to account for the few remaining winds of our climate.

Of these a north wind is perhaps the most rare; it is seldom known to blow for a day or two except during the summer months, and is always attended with the most intense heat.

A south east wind is sometimes experienced, whose source is the air that accompanies the Gulf Stream; it is remarkable for the disagreeable sensations it produces.

334. The force of the wind may be measured by instruments called *Wind-Gauges*. The best of these is composed of a bent tube, in which a liquid is placed; at the upper extremity of one of its branches, it is bent at right angles into a horizontal aperture; this aperture is presented to the direction of the wind that enters it, and thus acts on the surface of the fluid contained in that branch of the tube; it is consequently depressed, and the

force of the wind is estimated by the difference of the level at which the liquid stands in the two branches of the tube.

> The greatest estimated velocity of the wind, is about 100 miles per hour; and with this it is capable of producing the most violent effects. See Cavallo, Vol. III.

335. A wind frequently begins to blow at that point of the extent of its influence that is farthest from the quarter whence its source is derived; thus, the N. E. storms that are felt along the seaboard of the United States, have been observed to set in several hours earlier in Philadelphia than in Boston. Franklin, Vol. III.

OF RAIN.

336. Water, which at the temperature of 212° is converted by farther increments of heat into an elastic fluid, is, at lower temperatures, attracted by the air, and chemically combined with it.

> The truth of this may be demonstrated by a simple experiment; for if a drop of water be enclosed in a phial of dry air, it will speedily disappear. If now, a portion of deliquescent salts be introduced, they will attract the water, and become moist. If the temperature of the phial be lowered, the air will become less transparent, and a film of moisture will settle upon the glass. Therefore, in this way we may account for the constant evaporation that takes place from the surface of the earth, and more particularly from the ocean.

337. Where water is chemically combined with air, it does not lessen its transparency. Nor can it be separated from it but by a change of temperature, or by substances that attract it powerfully. When separated by either of these causes, the nascent precipitation is marked by a loss of transparency, or cloudiness.

338. The capacity of air for moisture increases in a greater ratio than that in which the temperature increases. By the experi-

ments of Saussure, it would appear, that while the temperature increases in arithmetical progression, the quantity of moisture it is capable of holding in solution increases in geometrical.

Hence, if two portions of air, of different temperatures, and both saturated with humidity, be mixed together, a precipitation of moisture must take place. PLAYFAIR, § 419, § 420.

339. Upon this fact, Dr. Hutton of Edinburgh was led to found a very ingenious theory of the formation of clouds. If large portions of the atmosphere, saturated or nearly saturated with humidity, be driven against each other by contrary winds, he infers that a precipitation of humidity, or the formation of clouds, must take place.

This theory may be illustrated by a geometrical diagram. See PLAYFAIR, § 420, and 421. HUTTON, *Dissertations.*

340. Although clouds may be formed, and precipitation ensue, at any altitude, yet these phenomena are likely to occur most frequently near the plane which separates the two great currents spoken of in § 328. This stratum of air, elevated about 18,000 feet above the level of the sea, is hence justly called the region of the clouds.

When clouds are formed, they descend by the action of their gravity. If in descending, they meet with strata of dry and warm air, they are again dissolved and disappear; thus clouds are frequently seen to roll from the summits, down the sides of mountains, to a certain level, and then disappear. This phenomenon is considered by the neighbouring inhabitants as a sign of dry weather. When, instead of meeting with dry, they pass through moist strata of air, they are no longer absorbed; and as they come from high and cold regions of the atmosphere, they absorb heat from the lower strata to such an extent as to cause precipitation, and the moisture therein contained joins the descending cloud to increase the quantity of rain it discharges. For this reason, we often find the same rain more violent in valleys than on the neighbouring mountains.

341. Where two masses of air, of high, but different temperatures, are mixed together, the precipitation is greater than when two

equal masses, at lower temperatures, are mixed, although the difference of temperature is in both cases equal.

We may in this way account for the great quantity of rain that often falls in a small space of time in tropical climates; and for the violence of the showers that accompany thunder gusts in summer.

342. When precipitation of humidity takes place, where the temperature is lower than the freezing point, the particles are frozen, and, arranging themselves in crystalline masses, form flakes of snow.

These flakes sometimes exhibit themselves in the form of six-pointed stars.

343. It has been said, that when drops have once been formed, and are afterwards frozen in their descent, they constitute hail. See Playfair, § 429.

But as hail often takes place in the warmest weather, this cannot be considered as a sufficient explanation. The following theory of hail, taught for many years in Columbia College by my predecessor Dr. Kemp, is more consistent with the phenomena.

It is known that when water is frozen in a Torricellian vacuum, it granulates and assumes the form of hail; hail also reaches the ground with a very great velocity: hence we may conclude, that it is formed in very rare air, and in a high region of the atmosphere. The decomposition of organic substances, is constantly giving out hydrogen gas, and this, from its specific levity, rises to the higher regions of the atmosphere; here, as no gas can remain long over another unmixed, it mingles with atmospheric air, and becomes susceptible of being inflamed by electricity. Should it be thus acted upon, it forms water, will be condensed into a space much less than it formerly occupied, and would leave a vacuum, did not the adjacent portions of air rush in to fill the void. The sudden rarefaction of this air, will produce an intense cold; the newly formed water will be frozen, and under circumstances that will cause it to granulate; descending from a lofty region, it will have great velocity; formed from hydrogen gas, and by the electric discharge, it will occur most frequently during the summer months, and accompany lightning.

The same theory was also extended by its author, to the explanation of the cause of meteoric stones. In the absence of all certain knowledge, it would be unphilosophical to insist upon it; yet if water is held in solution by the air, it may carry with it such earthy substances and metallic oxides as are soluble in it; and if the metals, their combinations or their ultimate elements, be capable of existing in an ærial form, they may float in the atmosphere, and may be united into solid masses by electric or galvanic action.

A similar theory, having reference also to the agency of terrestrial volcanoes, is mentioned by PLAYFAIR, § 431.

344. In the absence of the sun, the earth becomes cooler than the superincumbent air. That stratum of air which is adjacent, will communicate to it a portion of its heat; the moisture it contains will be separated, form a cloud, and finally precipitate; the same action will be propagated by it to the next higher stratum; and thus a cloudiness or mist will be formed, extending to some distance from the earth. When the sun again appears, such part of the moisture as has not actually reached the ground, will be again rapidly absorbed.

In this manner, we may account for the formation of dew; which appears only in calm weather, and during the night. We also see why the mist that attends dew appears to rise, although the moisture actually falls.

345. The pressure of moisture in the atmosphere, is known by the deliquescence of certain salts; by the torsion of hempen and flaxen cords, of cat-gut, whalebone, and hair. These substances are called Hygrometric substances, and have been applied in various ways to the construction of instruments called Hygrometers. The most ingenious of these, are the Hygrometers of SAUSSURE, DELUC, and KATER.

Professor Leslie of Edinburgh, has proposed the use of a modification of his differential thermometer, as a Hygrometer; and this, of all others, is the least liable to exception.

346. The quantity of rain that falls in any one place, may be measured by means of an instrument that is called a rain-gauge.

The principle of this gauge, consists merely in its collecting all the water that falls upon a determinate surface, and then measuring its depth in inches and decimals. The best rain-gauge is that of Howard.

The quantity of rain varies in different places very considerably, and is subject to no fixed law. It is consequently needless to cite any particular instances.

347. That phenomenon, the water-spout, is supposed to be caused by the action of conflicting currents of wind, that form a whirl, in the interior of which is a vacuum, where the water rises, as in the common pump. Others suppose the phenomenon to be electrical. See FRANKLIN, *Works*, Vol. III. MUSCHENBROCK, Vol. II.

348. The intimate connexion between rain and the winds, is evident from many appearances.

(1) Where trade or other winds blow uniformly, hardly any rain falls; when the monsoons change, heavy rains take place.

(2) In tropical climates, the rainy season always takes place when the sun approaches the zenith, and at this time the winds are most variable.

(3) There are some spots of continual rain, and in them there seem to be opposite currents of air continually meeting.

(4) In other places, it hardly ever rains. They are usually far inland, and without any of those inequalities of surface that promote the mixture of air.

(5) In countries near the ocean, where the air is much loaded with moisture, every change of wind is accompanied by rain; this is remarkably the case in Scotland, and on the north western coast of England.

(6) Lowness of the barometer is a sign of rain; it indicates that the equilibrium of the atmosphere is destroyed, and it is therefore probable that, before it is restored, winds from different quarters, and at different temperatures, must come into collision with each other. See PLAYFAIR, § 423.

OF AIR AS THE VEHICLE OF SOUND.

349. When an elastic body is drawn from its natural position, and then permitted to return, it regains a state of rest by a series of isochronous oscillations. These vibrations being communicated to the air, that is also a compressible and elastic body, produce therein alternate condensations and dilations, that are gradually propagated throughout the whole mass, in the same way that waves are formed upon the surface of a fluid. When these waves succeed each other rapidly, they cause in the organ of hearing a sensation that is called SOUND. BIOT, Vol. II.

In order to show that sound may be caused by the rapid oscillation of an elastic body, take a long and thin plate of steel, one of whose extremities is fixed in a vice, if it be drawn to one side and then left to itself, a series of vibrations will begin; if the plate be long enough, these vibrations may be counted, but in that case, they will produce no sound; if the plate be now shortened, the vibrations will become more frequent, and after they can no longer be counted, a sound begins to be heard. If we have counted the vibrations of the longer plate, we may now calculate the number of vibrations that the sounding body makes, on the principle that the number of vibrations are inversely as the squares of the lengths of the vibrating bodies. BIOT, liv. II.

To show that air is a vehicle of sound, we have recourse to the experiment of the bell in an exhausted receiver.

350. Air is not the only vehicle of sound; the other elastic fluids, whether gases or vapour, conduct it also. Liquids also convey sound, and so do solid bodies. BIOT, liv. II.

That water is capable of transmitting sound, might be inferred from the well-known fact, that fishes are furnished with organs of hearing. We may however prove it fully, by ringing a bell beneath the surface of the water.

351. The transmission of sound through the air, is not instantaneous, but it requires a certain time to travel from one

place to another. By the investigations of Newton, it would appear that the velocity of sound is 968 feet per second, while actual experiment makes it 1142 feet.

The correctness and accuracy of Newton's investigation, as far as mathematics is alone concerned, has been fully tested by succeeding analysts. It, therefore, is important to ascertain the circumstances that have not been taken into account, and by the omission of which, the mathematical process is rendered erroneous. Newton himself supposes the atmosphere to contain sulphureous and saline particles, but modern chemistry has shown that these do not exist. Neither does the rapidity of the oscillation bring the particles of the air itself into contact. Some have supposed, that the presence of moisture in the atmosphere, would be sufficient to account for the acceleration of sound; but if the air were saturated with moisture, the increased velocity would not be 8 feet per second.

Another probable cause, was pointed out by Laplace, viz. the action of the heat that is let loose by the compression of the air. This has been fully investigated by Biot, and is shown by him to be sufficient to explain the phenomenon. Biot, liv. II.

352. The velocity with which sound is propagated in a liquid, is greater than in a gaseous body; its intensity is also greater; but this is much diminished when the sound passes from the water into the air, and vice versa.

The motion of sound is much more rapid in solid than in fluid bodies; but although experiments have been contrived for the purpose of determining the actual rate, they have not been entirely successful. Biot however states, that sound travels more than ten times as fast through cast iron as it does through the air. Biot, liv. II.

The best experiments on these subjects, have been made by Perolle and Chladni. Chladni, *Acoustique*, § 217.

353. The great variety of sounds that the human ear is capable of distinguishing arises from three causes. 1. The greater or less frequency of the vibrations of sonorous bodies; 2. From

the greater or less intensity ; and, 3. From the greater or less simplicity of the sound.

Sounds are consequently distinguished, as high or low, acute or grave, flat or sharp, as loud or weak, and as clear or confused, &c.

354. Sound being conveyed through the air in a succession of waves, is capable of being reflected by obstacles, in which, every point of incidence becomes a new point of propagation. From a knowledge of this fact, we may derive the explanation of many curious phenomena.

Where the obstacle is near the sonorous body, the reflected sound reaches the ear at so short an interval after the original one, that they appear as one, but increased in intensity. In this way, we see the use of sounding boards in musical instruments. In the Italian Theatres, the side of the orchestra is made one great sounding board.

Where the obstacle is more remote, the sound first becomes confused, and finally we hear the reflected sound separate and distinct from the other. It is then called an echo. The ear is capable of distinguishing between 8 successive sounds in a second of time ; hence, if the reflected follow the original sound at an interval of more than 140 feet an echo is produced.

The same voice appears louder in a close room than in the open air; but if the apartment be large, confusion will often arise from the reflected sounds. This confusion may be much increased by an improper construction.

It is of the utmost importance that Churches, the Halls of Legislative Assemblies, Theatres, Concert Rooms, &c. should be constructed upon proper principles ; so that the voice may be as much aided as possible, by the reflection of the sound, and yet no disturbance or echo take place.

355. Where the waves that proceed from a sonorous body strike against continuous obstacles, and are reflected in such a manner as to pass through one or more points, either together, or at small intervals of time, a sound will be heard by an ear placed there, that will often be louder than the original sound.

In an ellipsoid, for instance, if a sound be uttered in one of the foci, all the reflected waves will cross one another at the same instant in the other focus. A conversation in a low tone, may therefore be kept up in a room of this shape, between two persons placed in the two foci, that will be inaudible to any other persons.

To produce this effect, the ellipsoid need neither be very large nor continuous.

In a spherical figure, the same effect will be produced in relation to two points equi-distant from the centre, and situated in the same diameter; for the interval will not be sufficient to render the sound confused. In this way, we may explain the phenomena of whispering galleries, and account for many deceptions that have been practised on the credulity of mankind. Others have, however, been practised, by making use of the rapidity with which solid bodies conduct sound.

The forms that are best fitted for producing such phenomena, are the worst possible for those of distinct hearing; such forms should therefore be always avoided. Mere theory would have shown that the representative chamber at Washington, and the Theatre at New-York, would be, as experience has proved them, almost totally unfit for their intended objects. In both these buildings, the roofs are vaulted and reverberate the sounds; in the latter, there is actually a good whispering gallery. The reader who wishes to pursue this subject farther, may consult CHLADNI, *Acoustique*, § 210.

356. The human voice may be rendered audible at great distances, by means of the speaking trumpet. To produce this effect, the waves that would otherwise be dispersed through the air, must be reflected from the sides of the tube, and reinforce the sound in the direction of the axis. A cylindrical tube will not do this; but it is necessary that the tube should enlarge at the extremity most distant from the mouth. CHLADNI, § 201.

Some have supposed, that the speaking trumpet acted by the communication of motion to a series of elastic bodies; as the quantity of motion communicated by this means, is greatest, when the bodies are in geometric progression, the logarithmic curve has been recommended as a proper one to form the outline of the speaking trumpet. HELSHAM, *Lecture*.

357. The hearing trumpet is the reverse of the speaking trumpet, and is so contrived, that all the sound that falls upon a large surface, may be concentered in the auditory canal of persons whose hearing is defective. A variety of figures has been given to this instrument, hollow cones, paraboloids, and ellipsoids, likewise figures the concavity of whose section is turned towards the axis. In some of them, for convenience sake, the instrument is twisted.

Of Musical Sounds.

A musical sound is one whose vibrations can be estimated.

An interval is the relation which the number of vibrations of two musical sounds have to each other. We usually take a grave sound as the standard with which we compare one that is more acute. Melody is a succession of musical sounds.

Two musical sounds expressed at the same time constitute an accord. Harmony is a succession of accords, or the co-existence of several melodies. Chladni, § 4.

358. In the lowest sounds that are perceptible to the human ear, the sounding body makes at least 30 vibrations per second; and it is still capable of receiving the impression where the sounding body makes from 8 to 12000 vibrations.

Chladni takes for his *ut* or fundamental note, one whose number of vibrations per second, is some power of the number 2. The lowest note of the violoncello or piano forte, he considers as caused by 128 vibrations of the sounding body per second. From this, the number of vibrations that cause any other musical sound whatsoever may be determined by calculation. Chladni, § 5.

359. The greater or less simplicity that exists, in the relation between the numbers of the vibrations of sounding bodies, is the source of all harmony.

An interval is a concord where this relation is simple, a discord where it is complex. The ear, although unable to count the number of vibrations, is capable of observing the effect of such vibrations as return together. Chladni, § 6.

The nicety with which an ear perceives the coincidence, causes it sometimes to be called a musical ear, or an ear for music.

360. The simplest of all relations, is that of 1 : 1, or where all the vibrations of two sounding bodies are performed in equal times. This relation is called the *Unison.*

361. The interval where the number of the vibrations of one body is double that of the other is called the octave. It is so named, because it is the eighth note of the musical scale, and every other note in the scale takes its name from the number of the place it occupies.

It is well known from experiment, that a sound and its octave have such a resemblance, that one may be considered as a repetition of the other. Hence,

(1) The nature of interval is not affected by taking the sounds that compose it one or more octaves higher or lower :

(2) All possible intervals may be regarded as comprised within a single octave, and may therefore be expressed by improper fractions, not less than 1, nor larger than 2.

362. The interval where the number of vibrations of one body is triple that of the other, is the next in simplicity to the octave. Where it is regarded as belonging to the first octave, on the above principle, its fractional form $\frac{3}{1}$ becomes $\frac{3}{2}$. It is then the fifth to the fundamental note. The next relation in simplicity, is that of $\frac{4}{1}$; but this is the second octave to the fundamental note, and does not extend the scale. $\frac{5}{1}$ is the next interval, and when reduced to the first octave, it becomes $\frac{5}{4}$. This expresses the interval that is called the third.

A note with its third and fifth, is called a perfect accord.

At the origin of music, its scale appears to have been limited to these three notes ; but they do not constitute a scale in consequence of the great extent and inequality of the intervals. The scale is completed by adding,

(1) Two notes that constitute a perfect accord with the fifth, as follows, viz.

$$\tfrac{3}{2} \times \tfrac{5}{4} = \tfrac{15}{8} \quad \tfrac{3}{2} \times \tfrac{3}{2} = \tfrac{4}{9}$$

and

(2) Two notes, such that they shall form another perfect accord, of which the octave becomes the fifth, viz.

$$1 : \tfrac{5}{4} : \tfrac{3}{2} : : \tfrac{4}{3} : \tfrac{5}{3} : 2$$

In this manner, we obtain the musical scale of 7 notes. Besides the names drawn from their numbers, they are known by particular appellations in some countries, and by letters in others; the intervals, with their several distinctive epithets, are as follows, viz.

1	$\tfrac{9}{8}$,	$\tfrac{5}{4}$,	$\tfrac{4}{3}$,	$\tfrac{3}{2}$,	$\tfrac{5}{3}$,	$\tfrac{15}{8}$,	2
1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.
C	D	E	F	G	A	B	C
ut	*re*	*mi*	*fa*	*sol*	*la*	*si*	*ut*

This scale is called the diatonic scale. CHLADNI, § 12.

It is also called the scale of the major mode, because, in addition to the perfect accord we have made use of, that is called *Major*, there is another composed of the intervals $1 : \tfrac{6}{5} : \tfrac{3}{2}$ that is called *Minor*. CHLADNI, § 9.

363. The ratios of the numbers and fractions, that express the intervals of the scale of the major mode, are of three different magnitudes, viz.

of 9 : 8, of 10 : 9 and of 16 : 15.

The first of these is called a major tone, and designated by the letter T; the second is called a minor tone, and designated *t*; the third a semitone, and designated H. They succeed one another in the following order:

T, *t*, H, T, *t*, T, H.

CAVALLO, Vol. II.

364. In the scale of the minor mode, they succeed one another in a different order, say,

T, H, *t*, H, T, *t*, T,

If, then, the sixth be considered as the fundamental note, this order of intervals may be expressed by the same sounds as the other; but in this different order of arrangement:

A	B	C	D	E	F	G
la	*si*	*ut*	*re*	*mi*	*fa*	*sol*

365. Nature appears to admit no other order of intervals that are fit for music. Therefore, in the diatonic scale, no other note than A or C, can be taken for the key or fundamental note. But the ingenuity of musicians has contrived to make every note of the diatonic scale capable of being the key note, either of the major or minor mode. Cavallo, Vol. III.

This object has been accomplished by the interposition of certain intervals or half notes between those of the diatonic scale. These are used occasionally, and have no appropriate individual name or letter, but derive their appellation from the neighbouring principal notes, with the addition of *Sharp* or *Flat*, according as their number of vibrations is greater or less than of those that produce the principal sound itself.

These half notes are interposed, to divide the intervals of the major and minor tones; they are consequently five in number:

C* D* F* G* A*

D^b E^b G^b A^b B^b

This method is not however perfect, as may be ascertained by taking D for the key note, and comparing its number of vibrations with the notes that become its 3d and 5th. It is, however, well known by experience, that if an interval occur, that differs but little from one that may be expressed by simple numbers, it sounds to our ear almost exactly like the simpler. This is a happy circumstance, for it affords the means of obviating the difficulty, by dividing the error among several notes, instead of accumulating it upon one. Thus, in tuning instruments, we do not make either the major or minor mode of any one key perfect.

This mode of obviating the disadvantage, is called *Temperament*. For a more full view of this subject, see CHLADNI, § 21, et seq. and BIOT, Vol. II.

Of the Vibrations of Sounding Bodies.

366. The elasticity of a sonorous body is the cause of its vibrations; and this elasticity may arise either from tension, compression, or from the hardness of the body itself.

The direction of the vibrations of a sonorous body may be either longitudinal, transverse, or circular. CHLADNI, § 34.

In its tranverse vibrations, the parts of a sounding body pass alternately backwards and forwards, in lines perpendicular to the axis.

The longitudinal vibrations of a sounding body, consist in contractions and dilatations of the sounding body, or its parts, in the direction of its axis.

Circular vibrations are performed in a reciprocating manner. In cylindrical or prismatic rods, the sounds produced by this species of vibration are always a fifth lower than the sound produced by the longitudinal vibration of the same body.

The intensity of sound depends on the greater or less arc described for the vibrations, on the size of the sonorous body, and on the velocity of the vibrations. CHLADNI, § 35.

367. A musical string may either vibrate entire, or divided into a number of equal parts separated from each other, by points that are called *Nodes*.

The lowest sound that can be produced in this way, is that of the entire string. If it be divided into two parts, the sound is its octave; if into three, the 5th above its octave, &c.

The sounds of the aliquot parts of a string, may be produced by laying the finger lightly upon the proper node, and then applying the bow to the middle of the part that is to vibrate. Sounds of this sort are called harmonic sounds.

368. The vibrations of membranes stretched lengthwise differ but little from those of strings; the nodes of their vibra-

tions are, in this case, fixed transverse lines. Where the membrane is stretched in more than one direction, the investigation of the law of its vibration becomes more complex. CHLADNI, § 46.

369. The vibrations of the air that can be most readily investigated are such as are produced by its passage through narrow openings and passages. The velocity of these vibrations depends on the velocity of the current of air, and on the magnitude of the opening. CHLADNI, § 49.

370. If a rapid current of air, passing through a narrow opening, or pipe, strike against a membrane, and causes it to vibrate, the sound is much increased. But it also becomes rougher.

The voice of men and animals is produced upon the above principle. There are in the larynx two membranes of a figure nearly semicircular, that, when applied to each other, form a complete circular surface. The circumference of these membranes, that are called the ligaments of the glottis, is fastened to the sides of the larynx, and their straight edges are capable of either closing completely, or leaving a lenticular opening that is called the glottis.

Where this opening is large, the air passes without causing sound; this is the case in our ordinary respiration. But if the membranes be drawn together, the air that passes from the lungs by the larynx, acts upon the membranes; they are thus set in motion, and communicate their vibrations to the current of air that is passing out. The other organs of the mouth act upon this current, and each of them produces an effect upon the voice. The closer the glottis is drawn by the action of the ligaments, the more acute the sound.

Every possible variety of sound is produced by a change of the magnitude of the opening, within limits that do not exceed $\frac{1}{10}$ of an inch.

Those who wish to pursue the subject of the vibrations of sonorous bodies, may consult CHLADNI.

Of the Organ of Hearing.

Our organs of hearing are situated on the two opposite sides of the head, in cavities of the temporal bones. The outer ear is a cartilage of a form nearly that of half an ellipsis. It appears to be intended to strengthen the sound.

The auditory passage is partly cartilaginous, partly bony; it is closed by the tympanum, a membrane that receives the impression of the waves of air, in order to communicate them to the inner ear.

Between the tympanum and the labyrinth, is the cavity of the tympanum, an irregular space that differs but little from a hemisphere; it is filled with air, and communicates with the mouth, by means of the eustachian tube.

Beyond this, we find the two openings of the labyrinth, called the round and oval windows from their figure; the round window is closed by a membrane; through the oval, the impressions made upon the tympanum are conveyed to the labyrinth by means of a series of small bones; these are four in number, and are styled the anvil, the hammer, the lenticular bone, and the stirrup. The base of the latter closes the oval window. The labyrinth is so called, from its complicated figure and parts; it is the inner portion of the organ of hearing, and is closely enveloped in the temporal bone; it contains the substance of the auditory nerve, spread in membranes and fibres in a gelatinous liquid. The parts of which the labyrinth is composed, are three semicircular canals, the cochlea, and the vestibule.

371. The impression of sound is communicated in the following manner:

The vibrations of the air, caused by those of the sonorous body, strike upon the tympanum; this sets in motion the four small bones that act upon one another, like a combination of levers, and the base of the stirrup communicates the impression to the gelatinous liquid contained in the labyrinth. The vibrations of the tympanum are also communicated to the air in its cavity, and this transmits them to the round window; thus the impression is made at once, in two different ways. The auditory nerve, whose substance is spread throughout the labyrinth, transmits these impressions to the brain, the common centre of sensation. CHLADNI, § 232.

372. We also hear sounds propagated through liquid or solid substances, especially when the vibration is communicated to the solid parts of the head, that transmit it to the auditory nerve.

The impression received in this way is stronger than that communicated through the air, and conveyed to the ear in the common way. Chladni, § 233.

373. The sound of our own voice is conveyed to the inner ear by means of the eustachian tube ; and when the usual channel of sound is closed, we may often hear by opening the mouth.

MECHANICS.

APPENDIX I.—OF MACHINES.

374. Machines may be considered as tools interposed between the natural agent, or worker, and the task to be performed, to render that work capable of being executed, which would have been difficult, if not impossible, without the intervention of some such contrivance.

375. We interpose machines between the agent, and the work to be performed, for the following reasons, viz.

(1) To accommodate the direction of the moving power to that of the resistance.

(2) To render a power, that has a fixed and determinate velocity, efficacious in performing work with a given velocity.

(3) To enable a natural agent, having a given intensity, to overcome another power or obstacle, whose intensity of resistance is greater.

376. Compound machines consist of the mechanic powers combined with each other in various ways, and modified in various manners. Hachette divides the elementary parts, of which machines are composed, into ten series, having relation to the change they produce in the direction or nature of motion.

Of all the lines that a point can describe, upon a plane, the two simplest are the circle and the straight line. If the point describe an entire circle, turning continually in one direction, this species of motion is said to be *continuous circular;* if, after having described the entire circumference of the circle, or any portion of it, in one direction, the point shall return over the same path to its original position, the motion is said to be *reciprocating circular;* if it describe a straight line without changing its direction, the motion is *continuous rectilineal;* if it change its direction, and return over the same straight line, to the point whence it first set off, the motion is *reciprocating rectilineal.*

Among these four kinds of motion, there may take place ten different combinations; but of two of these, there is no practical instance; of the application of the remaining eight, examples will occur in the machines hereafter considered. See HACHETTE, *Traite Elementaire*, p. 5.

377. In every elementary machine, there are three motions to be considered, viz.

(1) That which results from the action of the moving power itself.

(2) The motion, whether circular or rectilineal, of that part of the machine which is immediately acted upon by the moving power.

(3) The motion that is transmitted, by means of this part of the machine. HACHETTE, p. 6.

378. Of the moving powers employed by mechanicians, the most important are, the force of gravity, the elasticity of springs, the strength of men and animals in motion, water, wind, the force of the elastic vapour of water or steam, and the expansive energy of the gases generated by the detonation of gunpowder.

The action of the three first may be investigated and explained upon the principles of dynamics; water, air, and steam, act according to laws that are determined by the principles of hydrodynamics; whence we also draw the laws that govern the action of the gases generated by fired gunpowder. This last, however, is limited in practice to one particular case, that has already been fully treated of.

Those machines, the investigation of whose action depends upon the properties of fluids, are called *Hydraulic Engines.*

To the above powers or mechanic agents may be added, the attractions of electricity, magnetism, and chemical affinity, and the variation in the pressure of the atmosphere, that is marked by the changes in the altitude of the mercury of the barometer.

OF THE MOTION OF MACHINES.

379. Where the momentum of the power applied to a machine is greater than the momentum of the resistance, the ma-

chine is put in motion; if the power be one that act more forcibly on bodies at rest than on bodies in motion, its action is of course diminished, and the acceleration of the machine is less in the second instance than in the first. The diminution of the accelerating force continues, till the instant when that force becomes equivalent to the resistance; the acceleration then ceases, and the motion of the machine becomes uniform. PLAYFAIR, § 185.

Gravity is the only force that is known to act with equal intensity on bodies whether at rest or in motion. In machines propelled by all other mechanic agents, a *permanent state* of working is soon induced.

By *Resistance*, we understand the resultant of all those forces that oppose themselves to the action of the moving power, whether arising from the action of gravity, the friction of the parts of the machine, the resistance of the medium, the work to be performed, or from any other cause whatsoever.

380. Where a machine attains its state of uniform motion, the momentum of the power is equal to that of the resistance, and is the same that would be in equilibrio with the resistance, were there no motion at all. PLAYFAIR, § 186.

This species of equilibrium is called dynamical.

381. In all machines, the work done is to be estimated, not from the quantity of resistance overcome, but from the quantity overcome in a given time; or it is as the momentum of resistance multiplied into its velocity. PLAYFAIR, § 187.

382. The strength of men and of animals is most powerful when directed against a fixed resistance; when the resistance is overcome, and the animal is in motion, its force is diminished; and with a certain velocity, the animal can do no work, but can only keep up the motion of its own body. PLAYFAIR, § 176.

If f represent the absolute effort of any moving force when it has no velocity, and W its velocity when it can do no work; and if F represent an effort corresponding to a given velocity, V, then, if the force be uniform :

$$F = f\left(1 - \frac{V}{W}\right)^2$$

Gregory, § 376.

383. In all machines that work with a uniform motion, there is a certain velocity, and a certain load, that give the greatest effect, and which are, therefore, more advantageous than any other. Playfair, § 188.

If the force be uniform, it will perform the greatest quantity of work, when the impelled point moves with one third of the greatest velocity of which it is capable. Gregory, § 377. Playfair, § 188.

If, therefore, the load that is just able to keep a machine acted upon by uniform force at rest, be found by experiment, it must be reduced to $\frac{4}{9}$ of this quantity, in order that the effect of the machine may be the greatest possible. Playfair, § 189. Gregory, § 377.

Although the force exerted by animals is not absolutely uniform, yet no great error will arise from considering it as such; the above theorems are therefore applicable to all machines moved by the strength of animals and of men.

384. It is of the utmost importance that the motion of every machine should be rendered as uniform as possible. To obtain this end, a fly or regulator is most commonly used.

385. A Fly is, generally speaking, a circular disk of metal, moving with great velocity. When the action of the accelerating force is more than equal to the resistance, the fly opposes its inertia, but gradually acquires an increased velocity, and corresponding momentum. When the action of the accelerating force becomes less, the fly does not at once lose its velocity, but parts with it gradually, distributing throughout the other parts of the excess of momentum it had previously acquired.

Although a fly adds in fact to the momentum of the resistance, yet it will frequently enable an irregular force to do work that it would otherwise be incapable of performing; thus, although a man who turns a winch is capable of exerting during a portion of its revolution a force equal to seventy pounds, yet as during another portion he can exert a force equal to no more than twenty-five, he cannot be employed to overcome a resistance greater than the latter; and thus the whole excess of his force is lost. By the application of a fly, he is enabled to work with a force equivalent to the raising of a weight of thirty pounds. GREGORY, Vol. II.

Two of the most remarkable instances of the use of the fly, are to be found in the steam engine, and in Vauloue's pile engine. The effect of a fly is proportioned to its velocity, when its weight and diameter are given.

Flies are sometimes constructed with leaves, but they are disadvantageous in working machines, for they waste the moving power unprofitably. When, however, the moving power is great, compared to the work to be performed; and where it acts with an accelerated force, as in the case of gravity, they may not be without their use. Thus they were formerly employed to regulate clocks, and are still employed in kitchen jacks.

386. Where the resistance is variable, as in the case where the same machine may be employed to perform very different quantities of work, the action of the moving power itself is regulated by an instrument called a *Governor.*

A governor consists of two heavy balls, suspended by a system of levers, forming a conical pendulum. Where the velocity is small, the balls hang close to the axis; as the velocity increases, they are carried out, their centrifugal force counteracting their gravity. The levers act upon a system of wheels and pinions; to which, where the balls are more distant than a certain quantity from the axis, they communicate motion in one direction; and when they are nearer, in the opposite. These wheels move machinery that acts to lessen the intensity of the moving power; thus, in water wheels, they close or open the aperture of the penstock; in wind mills, they increase or diminish the quantity of sail; and in steam engines, they open or shut a valve placed in the steam pipe.

387. A contrivance, acting upon the same principle as the fly, is sometimes used to accumulate or condense force received from a

long succession of impulses, and discharge it in an instant of time upon a given object.

We have an instance of this in the coining mill.

OF MACHINES MOVED BY THE FORCE OF GRAVITY.

388. When a weight in its descent causes the motion of a machine, it may sometimes be rendered uniform, by employing a resistance that increases with the velocity; when the velocity has reached a certain quantity, the retarding becomes equal to the accelerating force, and the velocity of the machine remains uniform.

A fly with leaves, such as has already been described, is a regulator of this sort.

389. Of those machines where gravity is the moving power, clocks are the most important. In them, a pendulum is employed as the regulator; this, by forcing the *Escapement* at each vibration against the last of a train of wheels and pinions put in motion by a weight, prevents their being accelerated by its descent, and, at the same time, receives an impulse just sufficient to continue its motion.

The exact adjustment of these two forces is a nice problem in practical mechanics. See REES' *Cyclopædia, Maintaining Power.*

390. In the clock, the velocity of the maintaining power is small, being no greater than that acquired by a falling body in a very short space of time; hence wheels are in it made to turn pinions, and velocity is gained at the expense of intensity of force. The calculation of the number of teeth on the wheels and pinions, and the diameters proper to transmit the maintaining power to the escapement, and thence to the pendulum, is abstruse. Modern artists, however, find this done to their hand, and content themselves with imitating the works of their predecessors.

As an instance of the manner in which this is effected, we subjoin an account of the parts of a common eight day clock. The first or *great wheel* carries the hour hand, and of course should revolve in twelve hours; it is attached by means of a ratchet wheel, to a barrel, around which a spiral groove of sixteen threads is cut to receive the cord by which the weight is suspended. The action of the ratchet wheel is such, that the weight in its descent moving the barrel, must carry the great wheel round with it, but when the clock is wound up by raising the weight, the barrel moves freely, leaving the great wheel at rest. The great wheel is cut into ninety-six teeth, and turns a pinion of eight teeth, on the same axis with which is the hour wheel, of sixty-four teeth, that turns a pinion of eight teeth; on the same axis with this, is the second wheel, that has sixty teeth, and turns a pinion of eight teeth; this pinion is upon the same axis with the swing wheel, and its plane is, in the common clock, perpendicular to that of the three other wheels; the swing wheel, in this case, is of that sort called a crown wheel; it has sixty teeth, and acts upon two pallets, by means of which a series of alternating impulses are communicated to the pendulum. The number of these impulses during a revolution of the great wheel will be, by § 66.

$$\frac{96}{8} \times \frac{64}{8} \times \frac{60}{8} \times 60 \times 2 = 84600$$

And in one hour $\dfrac{84600}{12} = 7200,$

or one in every half second. This clock may therefore be regulated by a half second pendulum.

A crown wheel and pallets, are among the earliest escapements, and by no means the best; they are however continually to be met with even in modern clocks. In the best clocks, particularly when intended for astronomical purposes, we find a variety of escapements, contrived with much ingenuity. For an account of which, see Rees' *Cyclopædia*, article *Escapement.*

The other improvements consist in applying a compensation pendulum, and a spring to continue the motion of the train while the clock is wound up. Much, too, appears to be gained in accuracy of workmanship, by the great division of labour. The several parts of a clock require sixteen different persons for their manufacture. See Rees' *Cyclopædia*, article *Clock-making.*

Most clocks are furnished with an additional part, composed of machinery that strikes the hours. Much ingenuity has in many instances been shown in the contrivance of this, particularly when accompanied by chimes, &c. In general, however, the accuracy of a time-keeper is impaired by any load beyond the train we have already described.

An ingenious and very simple clock was contrived by Franklin, and this has been modified by Ferguson. See FERGUSON, *Select Exercises*, p. 1, et seq.

OF MACHINES PROPELLED BY SPRINGS.

391. After a spring has been coiled up, it begins to expand itself, and continues to do so with a retarded velocity, until its elasticity ceases to act. The retardation of the velocity is nearly uniform, or, in other words, the elasticity of a spring is proportional to its distance from a state of quiescence.

This property of springs appears to have been discovered by Dr. Hooke, but the discovery is also claimed by Huygens.

392. A spring may be made to act with uniform force, by lengthening the lever to which it is applied, in the same proportion as its tension becomes less; in this manner, the spring is applied in the common watch to produce a motion nearly isochronous; the spring is wound up in a cylindrical barrel, to which the end of a chain coiled around another barrel is attached; this last, which is called the fusee, is of the figure of a conoid generated by the revolution of an hyperbola around its assymptote.

393. A watch is a portable machine, that measures the portions of time. It differs from a clock in this respect, that the one can only be used when at rest, while the other may be carried from place to place. For this reason, the pendulum is not applied to it as a regulator; its construction is more difficult, therefore, than that of the clock, and has for nearly four centuries exercised the skill of the most ingenious mechanics; nor has it yet reached the ultimate point of perfection.

In the common watch, there are five wheels, viz. The great wheel upon the same axis with the fusee, and connected with it by a ratchet that allows the spring to be wound up without interfering with the motion of the train; the great wheel drives a pinion fixed to the axis of the centre wheel; the centre wheel revolves in an hour, and drives a pinion fixed upon the axis of the third wheel; the third wheel drives a pinion fixed to the axis of the contrate wheel; the contrate wheel, whose teeth in the common watch are perpendicular to its plane, drives a pinion whose axis carries the balance wheel; the balance wheel acts upon two pallets, and thus gives a reciprocating motion to the balance.

The action of the balance as a regulator is aided by a small spring, that accelerates its motion during the first part of its vibration, and retards it during the last.

The number of teeth in the wheels and pinions, varies in watches constructed by different makers; as an example of the mode of the adjusting it, take the following:

Time of going, 28 hours.

Number of turns of the spiral on the fusee, 7.

Great wheel 48 teeth,
Pin. 12 — 60 centre wheel.
Pin. 8 — 56 third wheel.
Pin. 7 — 80 contrate wheel.
Pin. 6 — 9 balance wheel.
2 pallets.

Number of beats per hour 14,400, or 4 per second.

Time of revolution of the contrate wheel, 1 minute.

The dial work, by which the hands that show the hours and minutes are carried, is as follows, viz.

A pinion, called the cannon pinion, is fitted tight upon the arbour of the centre wheel, so as to be carried around with it in the space of an hour. This pinion has a hollow cylindrical shaft, with a square head on which the minute hand is fitted, and drives a small wheel, on whose axis there is a pinion that drives a second wheel; this second wheel has the cylindrical shaft of the cannon pinion for its axle; the number of teeth in the wheels and pinions are so adjusted to each other, that this last wheel shall be twelve times as long in its revolution as the cannon pinion, and the hour hand is fitted to it.

In a watch whose train is planned like that we have described above, a hand that will mark seconds may be adapted to the axis of the contrate wheel, and the beats of the watch will divide each of them into four equal parts. Rees' *Cyclopœdia,* article Watch.

394. The elasticity of a spring is affected by alternations of heat and cold, hence the form of the fusee will not alone be sufficient to render each impulse communicated to the balance equal. The balance spring is also affected by the same causes. To compensate the irregularity arising from these sources, we have two several contrivances.

(1) A compensating curb, by which the effective length of the balance spring is increased or diminished, according to the temperature; this curb is composed of slips of two different metals, and acts upon the same principle with the gridiron pendulum. It was used by Harrison in his chronometer; but he himself pointed out a source of error in it, which led to the adoption of

(2) A compensation balance. In this, the balance itself is composed of two different metals; it is now employed in all known chronometers. The best compensation balances are those of Mudge and Arnold.

395. Even the best springs are not homogeneous throughout, and thus the law discovered by Hooke is not physically true; the irregularity of action arising from this cause, is met by adjusting the general shape of the fusee to each particular case, for which purpose a tool has been contrived. The friction, another source of irregularity, is diminished by bushing with jewels the pivots of those wheels whose motion is the most rapid.

396. It might be fairly expected, that a watch, whose movement was aided by jewelled pivots, the fusee adjusted to the slightest irregularities of the main spring, the action of heat and cold compensated by a curb applied to the balance spring, or by the counteracting influence of the parts of the balance itself, would be a perfect time-keeper. Another source of irregularity, how-

ever, remains in the unequal transmission of power, arising from there being alternately favourable and unfavourable positions of the teeth of the wheels and pinions; to this is to be added the irregular action of the old scapement, composed of a crown wheel and pallets.

A remedy for the unequal transmission of power was introduced by Harrison; it is called the *Remontoir*; this is a slender spring that is wound up eight times in a minute, and is applied to turn the contrate wheel; the preceding parts of the train (increased by the introduction of two additional wheels and pinions, together with a fly) having no other use but to wind up this spring.

The crown wheel has been entirely abandoned in the best watches, and instead of it, a variety of escapements have been introduced. The general principle of these, is, that they shall communicate no more than a momentary impulse to the balance, and that this shall be applied at that point where the momentum of the balance is a maximum. The balance itself is made so heavy, that the maintaining power is not sufficient to set it in motion, but merely, as in the pendulum clock, to restore the quantity that is lost at each vibration. It would be impossible even to mention the names of the several escapements. For a description of them, see Rees, article *Escapement*.

397. A watch whose motion is rendered nearly regular by the above-mentioned contrivance, is called a Chronometer; it may be employed for a variety of purposes in Astronomy, but is particularly useful in determining the longitude at sea.

The watch in its most improved state is one of the most noble instances of human ingenuity; it has for this reason been chosen by Paley as an apt illustration of his arguments in his *Natural Theology*.

The best chronometers that have yet been constructed, are those of Parkinson and Frodsham.

The principle of the division of labour is carried to a great length in the manufacture of watches; it is computed that the several parts of the works of a watch are the production of twelve different workmen, before they come into the hands of the person whose name is impressed on them as the maker; and that he employs twenty-one more in finishing and preparing it for sale. Rees, article *Watch*.

OF MACHINES MOVED BY THE STRENGTH OF ANIMALS.

398. The effect of a descending weight would be to raise an equal weight to the height whence itself fell; this moving power has, therefore, an independent and definite value in itself; in the case of all other mechanical agents, we estimate their value in some conventional manner, in the same way that we compare forces with one another, by the intervention of some conventional force that is taken as the unit.

Thus, the powers of men and horses have been compared with each other; and we estimate the force of water, wind, and steam, in terms of the latter.

399. The action of a man and horse can only be compared together, when their powers are exerted to produce the same effect, or when applied to draught. The greatest force of draught that a man can exert, may be taken at seventy pounds; his greatest velocity in walking, six feet per second, or a little more than four miles per hour.

If we apply to this the theorem of § 383, a man should work to the greatest advantage in drawing up a weight of 31$\frac{1}{3}$ pounds, and walking with a velocity of two feet per second. GREGORY, § 378.

Euler and Playfair make the maximum weight only sixty pounds, the most advantageous application of human force to draw twenty-seven pounds, with a velocity of two feet per second. PLAYFAIR, § 178. EULER on *Vessels*, Book IV.

400. The utmost strength of a horse is estimated as about equivalent to a weight of 420 pounds; his utmost velocity in walking, 10 feet per second; his maximum of power will therefore be to raise a weight of 186$\frac{2}{3}$ pounds, with a velocity of 3$\frac{1}{3}$ feet per second. GREGORY, § 378.

The strength of a horse is usually estimated as seven times that of a man.

The estimate is made by Hachette in another manner, viz.

401. A man working 10 hours per day, and continuing his labour throughout the year, can raise 10,000 French cubic feet of water to the height of a foot; or one hundred and eleven cubic metres of water, to the height of one metre. He therefore proposes to represent the strength of a man by the number 111. The strength of a horse he represents by the number 777. HACHETTE, § 14, and § 21.

402. When a man walks, his muscular strength is exerted to produce two different effects; to move his weight forward with a certain degree of velocity; and to raise it to a certain height. The first of these is applied to draught, the second to the support and carriage of burthens.

From the erect position of man, and the columnar structure of his body, his power of producing the second effect is much the greatest. Seventy pounds is the greatest weight he can draw over a pully, while he is capable of moving under any weight less than twice his own.

According to Playfair, the greatest effort of a man's strength in raising a weight, is when it is to that of his body as 3 to 4. A Roman soldier was loaded with a weight of 60 Roman pounds, and travelled at the rate of 5 miles per hour.

A French Grenadier is loaded with 56 French pounds, and the rate of his march is 3 miles per hour.

403. A horse, on the other hand, can support much less weight on his back than he is capable of drawing; and while a man can ascend nearly perpendicularly upwards, a horse cannot mount a plane inclined more than 45 degrees to the horizon.

Of the Crane.

404. The crane is a machine made use of to raise heavy weights, and to remove them to a small distance from the place whence they are raised. The principal parts of a crane, are the arbour, the gib, and a wheel and axle. A rope, that is wound

around the axle, passes over a pulley at the extremity of the gib to the weight; this pulley is generally single, but is in some cases compound. The whole machine has a horizontal motion around the arbour as an axis, and in some cranes, the upper block of the pulley has a motion along the gib; by the combination of these two motions, a weight may be set down at any point within the area of circle, whose centre is in the axis of the arbour, and whose radius is the projection of the gib.

The axis of the wheel and axle of cranes is sometimes horizontal; they may then be moved by the muscular action of men, taking hold of pins placed on the circumference of the wheel, or by the weight of men walking within them; the wheels are sometimes cut into teeth, and propelled by pinions worked by a winch. At other times, the axis of the wheel and axle is vertical, and they are propelled in the manner of a capstan, by men pressing against bars, or moved by horses walking in a circular path.

Those wheels in which men walk are considered dangerous; this species of crane has been much improved by Albert, in France, and Whyte, in England. In their cranes, the men are placed upon a platform, at the height of the horizontal diameter of the wheel, and step upon leaves placed on its circumference.

This species of crane is called the *Footmill;* it has been lately introduced into the Penitentiary of New-York. It is an advantageous application of human strength; for, in the crane of Albert, the weight raised in a given time was to the whole estimated force of the men employed, as 8 to 10. No more, therefore, than $\frac{1}{5}$ of the power was lost, in consequence of the friction, &c. See HACHETTE, p. 240.

To enumerate the great variety of cranes employed in different cases, would far exceed our limits; the curious may consult HACHETTE, FERGUSON, and GREGORY.

Of the Pile Engine.

405. In the pile engine a heavy weight or ram is raised to a great height, and discharged at once upon the head of the pile that is to be driven. This machine is in some cases composed of a simple pulley, elevated upon posts that serve as guides to the ram; the cord of the pulley is divided into a number of

smaller ropes at the extremity, in order to permit a number of men to act at the same time. In another form of the machine, the cord of the pulley is wound around an axle, whose wheel is cut into teeth, and driven by a pinion; to this pinion, manual strength is applied by means of a winch. Such is the engine used in the harbour of New-York.

Where the pile engine is used in great works, it is propelled by horses; of this, we have an instance in the famous engine contrived by Vauloue, to drive the piles used in building Westminster Bridge. See FERGUSON.

Of the Venetian Machine.

406. The Venetian machine is a contrivance for removing the deposit made by water in the bottoms of harbours. It is named from the place in which it was first used; and from its great simplicity and excellence, has superseded, in most parts of Europe, all other contrivances for the purpose.

In the use of this machine, a large instrument, resembling a spade, and like it composed of a wooden handle and iron blade, is driven into the muddy deposit; this is effected by attaching it to the extremity of a horizontal beam like the working beam of a steam engine, whose opposite extremity is elevated by the force of a screw. A motion of the screw in the contrary direction withdraws the spade after the operation is completed. When the spade is driven to a proper depth, a scoop that moves around a pivot, traversing the handle of the spade just where it is inserted into the blade, is forced forward to meet the spade, by the joint action of a lever and a pulley, whose cord passes around a capstan.

This machine is worked by 5 men, and raises 60 cubic feet of mud every 5 minutes from a depth of 15 feet. HACHETTE, p. 250.

A very imperfect and inefficient machine is used for a similar purpose in the harbour of New-York.

Of Wheel Carriages.

407. Where the strength of men or of animals is applied to draught, the effect that has been stated is the weight that spe-

cies of exertion is capable of raising perpendicularly upwards, as in the case of its being suspended by a cord passing over a pulley. The same application of strength may be rendered efficacious in removing much heavier weights along a horizontal plane. In this case, the resistance consists, not in the weight of the body to be moved, but in its *Friction.*

The principle upon which wheels and rollers act, in the removal of friction, has been already explained. See § 91.

408. Where wheels are considered, without reference to the animal force by which they are propelled, it would appear that the larger they are, the more advantage is to be derived from their use, whether we consider them as employed to diminish friction, to surmount obstacles, or to depress them. The only case where small wheels are more advantageous than large, is where the obstacle is to be removed.

409. The height of wheels is however limited by the direction of the draught. The power of draught is exerted most directly, and consequently most powerfully, when the line of draught is horizontal. Besides the loss arising from obliquity of action, where the draught is not horizontal, a part of the power of the horse is employed, when the axle is lower than his breast, in raising the weight from the ground, and, when it is higher, in increasing the friction.

A horse, in drawing, leans forward, bearing a portion of his weight upon the collar; hence, in order that the draught be horizontal, a wheel with an axle a little lower than his breast must be made use of.

An axle lower than the horse's breast, is also advantageous on hilly roads, where, in rising, a portion of the power must be exerted to overcome the gravity of the load, and where, in descending, its spontaneous velocity is to be checked. On level roads, however, it is said that the axle may be made a little higher than the breast of the horse with advantage.

410. When a horse leans forward to the draught, he acts not only by his muscular strength, but also by his weight. For this reason, in moving heavy loads, horses of a large breed are found

more valuable than smaller ones, even of equal or superior muscular force; oxen, also, are found of great value wherever the required velocity is small.

In carriages with two wheels, a portion of the weight rests upon the back of the animal, and, where this is not too great, adds to his effective force.

A load laid upon the back of a horse will often enable him to overcome a resistance his strength was before unequal to. In this way we may explain the curious fact, observed in carriages driven by postilions, that the mounted horse is always in the best order; this observation, which holds good in the artillery services of Europe, and in posting on that continent, is not found true in England; but in England the roads are so good, that the strength of the horses is employed in obtaining velocity, rather than in overcoming friction; and in this case, an additional weight is an encumbrance.

The advantage of carriages with two wheels, ceases when they are made so large as to require more than a single horse for their draught. And for similar reasons, it is disadvantageous to draw even a four wheeled carriage by horses harnessed *tandem*. This rule, however, does not apply to a level road, or where velocity is the object in view.

411. In four wheeled carriages, the fore wheels are made lower than the hind wheels, for the convenience of turning. In every other respect, this plan is disadvantageous.

Some ignorant persons imagine that high wheels help to push forward the lower ones, that precede them. And many drivers consider that it is best to load the fore wheels with more than a due proportion of the load. Both of these impressions are erroneous. Where heavy loads are to be carried a considerable distance, four wheeled carriages, drawn by a number of horses, are most advantageous; for, although the same number of horses would draw at least $\frac{1}{3}$ more in carts, yet each cart would require a driver; in short distances, where the drivers are advantageously employed in loading and unloading, carts are preferable. In Scotland, the horses are so trained, that 5 or 6 carts may be driven by a single person, and their agriculturists boast much of the great superiority of this method, and of the good construction of their carts. The cart used in the city of New-York, and its vicinity, is however still better than that of Scotland; its usual load is 14 cwt. over unequal ground.

412. When horses are harnessed abreast, it seldom happens that their strength is equal; it is very easy however, by apportioning the length of the lever on which each draws to his power, to place them upon an equality in this respect.

413. Wheels are usually made dishing; an advantage is said to be thus gained, for where the carriage inclines, the lower spoke becomes vertical, and thus has the greatest strength at the moment when the strain is greatest. The axles are also usually inclined downwards; but this practice appears to be vicious, for it increases the resistance.

414. In carriages that are loaded with heavy weights, the wheels should be made broad. This, although not attended with any advantage on hard or very smooth roads, is of great value in those which are soft or unequal. It acts also to preserve and even improve the roads.

Hence in England, all heavy wagons are required by law to have broad wheels. This law, which met with great opposition from the carriers, has now been found to be as much for their private interest as for the good of the roads. On many of the turnpikes of the United States, broad wheeled wagons pass toll free; and it is to be hoped that this valuable improvement will rapidly come into general use.

The friction rollers of Mr. Garnett, mentioned in § 92, have also been applied to the wheels of carriages.

415. Springs, that were originally introduced for the ease of the person carried, have been found a mechanical advantage. Upon rough and uneven roads, they act on the same principle with the *Fly*.

See, for a more full dissertation on this subject, FERGUSON's *Lectures*, with *Additions*, by BREWSTER.

416. In wheel carriages, the equability of motion is even of greater importance than in other machines; sudden checks to motion having a more pernicious effect on an animate than it can have on an inanimate body. Playfair, § 192.

Hence we may see the very great importance of good roads, and the necessity that they should be constructed upon scientific principles. In low countries, every exertion should be used to make the roads as nearly level as possible; in hilly countries, the ascent from the lowest to the highest points, over which the road must unavoidably pass, should be, if possible, made a regularly inclined plane of the least practicable angle.

In almost every case, it is better to avoid and go round a hill, than to pass over it; the effective length of a road is measured, not upon a level line, but by the number of turns made by the wheels that move upon it.

In some countries Rail-ways have been introduced, when the weight of the articles to be transported in one direction was very great; these are composed of two parallel inclined planes of cast iron, on which the wheels of the carriages move; they answer the double purpose of diminishing the friction and equalizing the resistance, hence a horse is capable of drawing 5 times the weight upon them that he could over a road of similar inclination. See Rees, *Railways.*

Of Animals considered as Machines.

417. Animals may themselves be considered as machines, planned by the Creator with consummate wisdom, and admirably adapted to the several states and circumstances in which they are destined to exist.

418. The motion of animals is produced by the action of their muscles upon the bones. Each several motion is effected by the joint operation of a pair of antagonist muscles called the flexor and extensor; by the union of two or more pair of these, and by the simultaneous action of those of different bones, every variety of position and attitude of which an animal is capable is produced.

MECHANICS.

APPENDIX II.—HYDRAULIC ENGINES.

OF MACHINES FOR RAISING WATER.

422. Machines for raising water may be divided into two classes: (1) those which are moved by the direct action of the water itself; and (2) those which are moved through the intervention of some machine, of which any natural agent may be the prime mover.

Among the first class are the following machines, viz.

423. The *Fountain of Hero;* this is a machine by which the motion of a column of water is transmitted to another column of the same liquid, through the intervention of a stratum of air that separates them. The principle on which it acts, may be generalized, and comprehend all machines in which the motion of one liquid is transmitted to another, through the intervention of a stratum of fluid, whether compressible or incompressible, of less specific gravity than either of the two liquids.

Hero's fountain in its original form consists of two vases; the one placed perpendicularly over the other, and both air tight. A pipe, open at its upper extremity, passes through the upper vase without communicating with it, and enters the lower, reaching nearly to its bottom. From the upper part of the lower vase, a second pipe proceeds that reaches nearly to the top of the upper. A third pipe is inserted into the top of the upper vase, and reaches within a small distance of its bottom.

To use the machine, a portion of water is placed in the upper vase, and in the lower enough to immerse the lower extremities of the first and third pipes; the several openings are then closed, and water is poured into the first pipe; this column, of the whole height of the instrument, presses upon the air contained in the upper parts of both vases, and would condense it, did it not act upon the water in the upper vase, and force it out in a jet through the third pipe.

The action continues, if water is supplied, until the level of the water in the upper vase falls below the lower end of the third pipe.

Hero's fountain is but little used in its original shape, except for purposes of mere amusement. It has however, when modified, been applied to several valuable practical purposes. For instance, Girard, in France, has constructed a modification of Argand's lamp, where the oil is raised to the wick by a contrivance acting on the principle of Hero's fountain. See HACHETTE, p. 43.

At Schemnitz in Hungary, water is raised from the bottom of a deep mine by a machine of this same species. Its principal parts are two strong vessels; one of these (A) is placed on a level or horizontal shaft carried in from the open ground; the other (B) at the bottom of the mine. Above the *level*, the ground rises rapidly into a mountain, and at a considerable height on its side is a spring; the water of this spring is made to act on the air enclosed in the vessel A, this communicates with the air in the upper part of the vessel B, and forces the water from it through a rising pipe to the horizontal shaft, where it is discharged upon the plain. When the vessel A becomes filled with water, and B with air, stop-cocks are opened, by which the first is discharged along the same shaft, and the second allowed to escape; the machine is then ready to recommence its action. These cocks were, in the original machine, opened by hand, but an ingenious apparatus has been contrived by Boswell, in England, by the aid of which the machine is rendered self-working. HACHETTE, § 83. GREGORY, Vol. II.

424. Two buckets united by a rope passing over a pully are sometimes used as an hydraulic engine. One of these buckets (A,) weighs, when full of water, more than the other, (B;) but when both are empty, B is the heavier. If, then, they are both at the same level in consequence of the middle of the rope resting on the summit of the pully, and if water be poured into them, A will soon preponderate, and, in descending, raise B with its contents until it reach the pully. If at this moment, a pin or some other contrivance be applied, by which the buckets are overturned and emptied, (the water from A running to waste, and

that from B passing into a reservoir,) B will preponderate, and, in its turn descending, will raise A to the original level.

If the buckets be made of the figure of conic frustums, open at the larger end, and if the points of suspension be placed so that their axis passes between the centres of gravity of a solid and of a hollow frustum, the equilibrium of the buckets when full, will be momentary, but when empty stable; they may therefore be emptied by the slightest force.

425. The *Hydraulic Ram*, (*Belier*) of Mongolfier, acts upon the following principles :

When water is running with a rapid current through a pipe or close channel, if the end at which the water issues be suddenly stopped, the water by its acquired impetus, will act upon the sides or circumference of the pipe, and endeavour to escape with a force proportioned to its quantity and velocity; it may thus be made to issue with great violence from a small aperture opened at or near the end that is closed. If, then, an ascending pipe be joined to the aperture, a portion of the water will ascend. If the machine be provided with proper valves to close it at the end, and to prevent the return of the water through the ascending pipe, the operation may be repeated in continued succession.

The action of the principal valve of the hydraulic ram may be thus explained; when a body is acted upon by a force, it does not instantly acquire the whole velocity the force is capable of communicating. Thus, when a rope is tied to a shell placed in a mortar, it is broken when the mortar is fired; and thus a weight that if it passed slowly over a thin plank or weak ice would infallibly break them, produces no such effect when it moves rapidly.

The hydraulic ram is composed of an inclined pipe, down which the water flows; a valve that is of such a weight that the water can raise it when it has communicated to it its whole impetus, but which does not float on still water; a second valve that closes when the first opens, and vice versa; an air vessel that communicates with the descending pipe by means of the second valve; and an ascending pipe opening upwards from the air vessel.

The action of the fluid produces an alternate rise and fall of the first valve; the motion of the second valve reciprocating with that of the first, it admits water into the air vessel; here the air is condensed, and by its spring forces the water in a continued stream through the rising pipe. HACHETTE, § 94.

426. The *Syphon of Venturi* acts upon the principle discovered by him, of the lateral communication of motion in fluids. In it water is admitted into a vessel, and allowed to escape from an orifice near its bottom, by a cylindrical adjutage; a syphon is introduced into the adjutage at the vena contracta, and its longer branch descends into a reservoir; through this syphon the water rises, and is discharged along with that flowing from the upper vessel. HACHETTE, p. 35.

Among the second class of engines for raising water, are,

427. The *Tympanum*, a machine much used by the ancients; in it, a hollow cylinder whose axis is horizontal, is divided into a number of compartments, by partitions extending from the axis to the circumference. The lower part of the wheel dips into the water, and holes are pierced near the circumference, into which the fluid is received during the revolution of the wheel; as the wheel revolves, the fluid passes along the inner circumference until it reaches the next following partition, over which it is carried to the centre of the circle, where it is discharged.

In the use of this instrument, the centre of gravity of the fluid raised does not move in the shortest possible line. De la Faye has proposed to improve this machine, by causing the water to rise in a straight line; he effects this by making the channel in which the water moves in the form of the involute of a circle.

In both forms of this machine, water cannot be raised higher than half the diameter of the wheel.

428. The *Persian Wheel;* this has buckets hung upon its circumference by pins, in such a way, that they will be perpendicular when acted upon by no other force than their own weight,

and that of the water they may contain. The water received by the immersion of these buckets into a reservoir when at the lowest point of their revolution, is discharged at the moment they reach the summit of the wheel, by causing them to strike against a pin that overturns them.

The velocity of this wheel must not be very great, otherwise the whole of the water raised by each bucket will not be discharged into the channel placed to receive it.

This wheel is liable to a great resistance, in consequence of the difficulty of pressing the empty buckets into the lower reservoir; but it may be avoided by placing a valve opening upwards in the bottom of each bucket.

429. *Vera's Pump*, called also the Hessian or Rope pump, is formed of an endless rope, that passes over two pulleys; one of these is placed in the reservoir whence the water is to be raised, the other at the place of discharge; a quantity of water adheres to the rope, when at the lowest point of its revolution, and it is compelled to part with it by a gentle pressure when it has reached its highest situation.

A machine of this sort, constructed by the inventor himself, drew water from a well ninety-five feet deep; and a single man could raise six gallons per minute. REES, article *Water*.

430. The *Hydraulic Cane* is a long tube fitted with a valve opening upwards; this valve is sometimes placed at the bottom and sometimes at the top of the tube; the lower extremity of the tube is immersed in water, and a rapid alternating rectilineal motion is given to it; when the valve is at the top of the tube, the water that has entered it during its depression, is supported, while it is drawn upwards, by the pressure of the atmosphere; when it is again depressed, more enters, and the action is continued until the fluid issues from the top of the tube; when the valve is at the lower part of the tube, it supports, during the motion upwards, the water that has entered during the time of descent.

431. *Vialon's Pump* is formed of two tubes, formed into screws, by wrapping them around the same cylinder, but in con-

trary directions; the lower end of each tube is furnished with a valve opening upwards; a rapid reciprocating motion around the axis of the cylinder is communicated by means of a winch; at each alternation of movement, the water acts upon one of the valves, which opens and permits it to enter, at the same time the other valve closes and retains the water that has entered through it; the two spirals are drawn together at top, and joined into one common tube, through which the water they have raised is discharged.

This machine, and the hydraulic cane, whose valve is at its lower extremity, are both constructed on the same principle. HACHETTE, § 129.

432. The *Centrifugal Pump* is composed of a number of tubes arranged upon the surface of a truncated cone, whose larger base is uppermost; these tubes are bent outwards at their upper extremity, and a circular channel placed beneath their orifices; the lower extremity of the tubes is placed in the water to be raised; when the machine has a rapid motion given it around the axis of the cone, the centrifugal force in the lower circular base of the cone becomes sufficient to overcome the gravity of the water, it rises, and is discharged at the upper ends of the tubes into the circular channel. HACHETTE, § 130.

433. The *Screw of Archimedes* is composed of a solid cylindrical shaft, surrounded by a hollow cylinder having the same axis; the threads of the screw are contained in the hollow space, between the outer and inner cylinder, and are generated by the motion of lines that are constantly perpendicular to the axis of the cylinders, and apply themselves to screws of the same thread, supposed to be wound around their surfaces; from three to four such screws are usually wrapped around the solid cylinder. The upper part of the machine is furnished with a winch, the lower with a gudgeon, and these rest upon a frame of wood.

In order to apply the screw of Archimedes to raise water, the lower end is immersed in the fluid, and the axis inclined to the horizon; the winch is then turned in a direction contrary to that of the motion of the point that is supposed to have generated the screws; the water is by this motion carried along the surfaces of the screws, and falls at the upper part of the machine into a trough prepared to receive it.

The elevation of the axis of the machine has a limit that depends on the radius of the inner surface of the hollow cylinder, the angle of inclination of the screw, and the distance between its threads. See HACHETTE, § 134, et seq.

434. The *Chain Pump* is composed of a cylindrical or square barrel, and an endless chain furnished with several pistons of the same figure with the barrel, and placed at small distances from each other, on the links of the chain; the chain is generally extended between two wheels, one below, the other at the top of the barrel, but the lower of these is sometimes omitted; these wheels have forks fixed on their circumference at such distances as will permit a piston to fall into each of the intervals, while the fork takes hold of the links of the chain; to set the machine in motion, the upper wheel is turned by means of a winch; the pistons fit close to the barrel at the lower extremity, and for a few feet upwards, above this the barrel becomes larger, and the pistons move freely. When the lower end of the barrel is immersed in water, and the chain is made to circulate by turning the wheel, each piston will carry a portion of water into the barrel, until at last the barrel is filled and the water runs over the top; the pistons succeeding each other in regular rotation, produce a constant stream as long as the pump is worked, and water supplied from beneath.

The chain pump, if tight, is attended with a great degree of friction. It is sometimes inclined, and is then found to raise a greater quantity of water to the same height. The chain pump placed vertically, is principally used on shipboard. The inclined chain pump is used in draining coffer dams, and in other places where hydraulic erections are going on beneath the level of the water.

435. The principle of the *Common Pump* has already been explained; the difference that exists among machines of this kind, arises in part from the construction of their valves; of these, there is a very great variety.

(1) The common valve; this is a frustum of a solid cone of wood that falls into a hollow frustum or socket, that it nearly fills up; to the top of the solid frustum is attached a piece of leather of a circular form, that is larger than its greater base, and thus covers the space between the hollow and solid cones; one side of the leather is firmly fastened to the hollow cone, and the valve plays around this side as upon a hinge.

(2) The butterfly valve is in two pieces, and the leather is fastened to a solid piece that crosses the socket in the direction of its diameter; the two parts of the valve moving on this as a hinge, have a motion that resembles the action of wings, hence the name.

(3) In the pyramidal valve, the socket is square; from its angles rise four pieces that meet at an apex like the edges of a pyramid; the valve is composed of four triangular parts that are leathered, and fastened down to the socket; it opens from the apex of the pyramid, and when open, the parts lie against the sides of the pump.

(4) The triangular valve; in using this, the barrel of the pump and the socket of the valve are made square; the valve is in two portions, of the figure of a right angled triangle, and the leather, which is square, is fastened down to a piece that crosses from angle to angle of the socket in the direction of its diagonal.

These four valves may also be constructed of metal as well as of wood and leather; in doing this, the parts are ground to fit each other, and for the flexibility of the leather, hinges of metal are employed. The remaining valves that we shall mention, are always constructed of metal.

(5) The conical valve; in this a solid frustum of metal is ground to fit a hollow one; as the first is raised, a passage is opened between them, and it is prevented from rising too high by a key that arrests its progress, and on its return, guides it back to its proper position.

(6) The spherical valve; this is a sphere of metal, hollow within; the socket is ground into the form of the zone of a sphere, and thus the valve adapts itself to its cavity in whatever direction it falls.

(7) The sphere is sometimes made very light, and to enable it to return, it is attached to a long rod of metal, at the lower extremity of which is a weight; this valve is less liable to be choked than any other.

The limit of the height to which a common pump will raise water, is about 34 feet, as has been already explained.

436. The *Lifting Pump* consists, like the common one, of a barrel and two valves; in it, however, the upper valve is fixed in the barrel, and it is the lower valve that moves in the piston. The piston is moved beneath the surface of the water by an iron rod of the form of a stirrup. When the piston is depressed the valve opens, a portion of water passes through it into the barrel; when the piston is drawn up it *lifts* the water upon it, and forces the air contained in the barrel through the upper valve; when the piston is again pressed down, the upper valve closes, prevents the admission of air, and more water is admitted through the lower valve; after a few strokes, the water will rise through the upper valve, be lifted at each elevation of the piston, and prevented from returning at each depression by the closing of the upper valve.

The power of this pump, or the height to which it will raise water, is limited only by the strength of the materials, and the intensity of the moving power. The common pump may be converted into an engine that shall combine the effect of the lifting pump with its own by making its piston work air tight in a collar of leather, and adapting an ascending pipe instead of a spout. See Rees, article *Water*.

The lifting pump may be made with a solid plunger; in this form it has two pipes that open into the upper part of the barrel; one admits water from the well when the piston descends; through the other, the water is lifted as the piston rises; both pipes are furnished with valves opening upwards, that prevent the return of the water.

437. The *Forcing Pump* has a solid piston that works in the barrel; at the bottom of the barrel is a valve opening upwards that permits the water to enter when the piston is drawn up, but prevents its return; near the bottom of the barrel is introduced an ascending pipe with a valve opening upwards; at each descent of the piston the water in the barrel is forced up this pipe.

In the lifting, and in the common pump, the water is raised during the ascent of the piston, in the forcing pump during its depression. A pipe is frequently adapted to the lower valve of the forcing pump, not exceeding in length the limit of the common pump, and in this way the effect of the two may be combined.

In the pump of De la Hire, the lifting and forcing pumps are combined, and thus water may be raised by a single piston both in its ascent and descent. In this pump the moving power is resisted by but half the friction that would oppose it were two barrels used.

The forcing pump that is most commonly used in England, is the invention of Sir Samuel Morland; its barrel is not ground smooth and the piston fitted tightly to it, but the piston consists of a solid plunger of a cylindrical form nearly as large as the barrel of the pump, which it does not touch at any point; this plunger works air tight in a collar of leather, and in descending diminishes the capacity of the barrel, while in rising it increases it.

This kind of forcing pump is preferable to any other. There is a little nicety to be observed in making and fitting the collar of leather; for which, see REES, article *Water*.

438. To equalize the discharge of water from pumps, an air vessel is adapted to the ascending pipe. This, which may be a close chamber of any figure, communicates with the ascending pipe at its lower end; at its upper end, a pipe is inserted, reaching down through the chamber nearly to its bottom. The action of the pump will force water into the air vessel; the air being thus condensed, will by its elasticity endeavour to regain its former space, and equalize all sudden motions in the pipe.

439. The *Fire Engine* consists of two pumps, that force water into one common air vessel; they are placed in a wooden cistern, and the whole mounted on wheels.

440. When the height to which water is to be raised is small, a pump may be constructed in which the piston does not require to be fitted closely into the barrel, nor are any leathers necessary. The barrel of this pump must be as long as the whole height to which the water is to be raised, and as much more as the length of the stroke of the piston. The piston is a solid piece of wood, fitted to the barrel as closely as it can be without actually touching; it must be as long as the barrel itself

so that when it is let down, it will occupy the whole interior of the barrel, except the small space that is left to prevent their surfaces from coming in contact. The bottom of the barrel has a valve opening upwards, and a pipe rises from its lower part to convey the water raised by the pump to the reservoir. This pipe has a valve opening upwards, to prevent the return of the water from the reservoir, whose altitude must be less than the height of the barrel of the pump.

When this pump is fixed for work, the lower end must be immersed as far into the water as is equal to the length of the stroke of the piston, so that the lower end of the piston may never rise above the level of the water. When the piston is drawn up, the water enters the barrel to the height at which it stands in the well by its own pressure; when the piston descends, it forces the chief part of this into the ascending pipe, while another portion rises between the piston and the sides of the pump.

441. Pumps are sometimes constructed on the principle of the common pump, but with a diaphragm of leather, in which the upper valve plays. These are subject to but little friction, and have no other fault but the liability of the leather to crack.

OF THE WATER PRESS.

442. If there be a mutual communication between two columns of any fluid, whatever pressure or effort may be exerted on the one, will be transmitted to the other in a ratio proportional to the respective area of each column. In this sense, as we have already seen, every fluid is in itself a machine.

On the above principle, Bramah, an ingenious artist of Great-Britain, has constructed a water press engine that goes by his name. This is now applied to a very great extent, in various departments of the useful arts

It consists of a small forcing pump, whose plunger works through a collar of leathers like the pump of Morland; the pipe of discharge, which has as usual a valve opening upwards, is carried along horizontally, and enters a cylinder, whose diameter is much greater than that of the pump; to this cylinder is adapted a piston that works in it water tight. When the plunger of the forcing pump is pressed down, water is forced into the cylinder, and acts upon the piston, on which it presses with a force as much greater than that which presses on the plunger of the pump as the area of the section of the former is greater than the area of the section of the latter. If this ratio be multiplied by the ratio of the arms of the lever, applied as a pump rod, to each other, we shall have the proportion between the power applied to work the press, and the pressure it is capable of producing. See HACHETTE, § 173. REES, article *Press.*

OF WATER WHEELS.

443. A circular motion may be produced either by the impulse, by the weight, or by the re-action of water.

Undershot wheels are examples of the first of these methods, Breast and Overshot wheels of the second, and Barker's mill of the third.

In the undershot wheel, the water acts upon leaves or float boards, passing beneath the wheel in a current. The properties of this machine have been investigated analytically by means of the hypothesis we have already employed, and very accurate experiments have been made on its action by Smeaton. We shall give the theoretic deductions, and state how far they have been found to agree with the experiment.

444. If a stream of water impinge upon the float boards of an undershot wheel, and escape from it the instant after it has made its impact, the quantity of water which actually strikes the wheel will be to the whole quantity that passes by it in a given time, as the relative velocity of the wheel to the absolute velocity of the water.

Hence, the force exerted by the water to turn the wheel, is as the square of the relative velocity of the wheel. GREGORY, § 469.

445. The effect produced by an undershot wheel, is a maximum when its absolute velocity is one third of the velocity of the stream. GREGORY, § 470.

By experiment, this is found to vary between $\frac{1}{3}$ and $\frac{1}{2}$; the reason of this is, that the water does not escape immediately after impact, but is confined by the channel.

446. If the velocity of the stream be given, the effect is as the quantity of water expended.

This is fully confirmed by the experiments of Smeaton, as are also the following corollaries:

(1) Where the expense of the water is the same, the effect is as the square of the velocity.

(2) The expense of water being the same, the effect is as the height of the head of water, or rather of the effective head.

(3) The aperture being the same, the effect will be as the cube of the velocity. GREGORY, § 471. SMEATON, *Experimental Inquiry*, p. 15, et seq. PHILOSOPHICAL TRANSACTIONS, for 1759.

It was formerly believed that the floats on an undershot wheel should be so arranged, that when one was perpendicular and fully immersed in the stream, the preceding float should be just leaving, and the following one just entering the water. By Smeaton's experiments, it would appear that this is not true; he found that, in his apparatus, power was lost when the number of floats was diminished below 24, and that no power was gained by increasing that number. With 24 floats 3 are constantly immersed, and this appears to be the rule best adapted for practice. SMEATON, p. 24.

The effect produced by an undershot wheel is to the power as 3 to 10. SMEATON, p. 23.

447. In the overshot wheel, the water is received in buckets placed upon the circumference where it acts by its gravity; it is sometimes admitted at the top of the wheel without exerting any impulsive force, at other times it is permitted to act by its impetus. In the breast wheel, only half the buckets in a semi-circumference, or even fewer, are filled with water, it being admitted at the extremity of, or beneath the horizontal diameter.

448. The effective weight of water in an overshot wheel, may be computed by multiplying the quantity in each bucket by the sine of the angle which the diameter, passing through the centre of gravity, makes with the vertical diameter. GREGORY, § 474.

If the number of buckets were 12, and all supposed equally full, the real weight of the water would be to its effective weight as 1 to .622008.

If the number of buckets were 24, the real weight would be to the effective as 1 to .632214.

If the number of buckets be so increased that the weight may be considered as diffused equally over every portion of the semi-circumference, the real is to the effective weight as 1 to .63662. GREGORY, § 474.

449. Supposing the water to act partly by impulse, and partly by its weight, an overshot wheel will be, theoretically, in its greatest perfection when its height is two-thirds of the head of water.

In practice, many inconveniences are found to result from admitting the water to act by its impulse; and it is ascertained by experiment, to be best to make the height of wheel nearly equal to that of the fall, and to admit the water with a velocity but little greater than that of the circumference of the wheel. SMEATON, p. 31.

450. The efficacy of an overshot wheel is to that of an undershot wheel as 13 to 5, when the head of water, the aperture, and the diameter of the wheels, are equal. GREGORY, § 476.

This theoretic deduction is not confirmed by experiment; Smeaton makes the proportion between the two effects, as 2 to 1. SMEATON, p. 30.

451. The more slow the motion of an overshot wheel, the greater will be its power; for the accumulation of water in the wheels more than counterbalances the diminution of the velocity; but there is found a practical limit to this, for if the velocity of the circumference become less than from 2 to 3 feet per second, the wheel is no longer steady, and is liable to irregularities. SMEATON, p. 35.

452. Where a circular motion is produced by the re-action of water, the machine is called *Barker's Mill*, after the name of the inventor. In Barker's mill, the water is introduced through a cylinder moveable about a vertical axis into two horizontal boxes that communicate with it. The boxes have openings at the sides in opposite directions.

> When the apertures are shut, and the whole filled with water, the machine is in equilibrio ; but if the apertures are opened, the columns having these apertures for bases, will cease to press on the sides, the equilibrium will be disturbed, and a rotary motion take place in a direction opposite to that in which the water issues. PLAYFAIR, § 306.

453. If A be the sum of the areas of the two orifices, and *h* the height of the vertical tube, the theoretic effect of Barker's mill will be A *h*, or equal to the whole momentum of the water expended. It would therefore appear to be the most advantageous application of a fall of water. In practice it does not answer the expectations that might be formed from the theory ; it is liable to great friction, and a centrifugal motion takes place in the vertical pipe, that diminishes the pressure of the head.

> Mr. Rumsey, an American, has proposed to remove this last inconvenience, by introducing the water through a bent tube from beneath the boxes.

454. A variety of other ingenious contrivances have been proposed for the purpose of applying water as a moving power. None, however, have come into general use, wherefore it is unnecessary to mention them here.

455. If a rotary motion be given to a breast wheel by a windmill, a steam engine, or any other mechanic power, it may be applied to raise water from one level to another. It is then called the *Fen* or *Flash Wheel*

OF VESSELS.

The most important application of the theory of the motion of bodies in fluids, is to the subject of vessels intended for navigation. In the consideration of this question, we shall follow for the most part, EULER's *Complete Theory of the Construction and Properties of Vessels.*

456. Whatever difference there may be in the construction of vessels used in navigation, we always find in them this one general property, that each vessel is composed of two perfectly equal parts joined together in the middle, lengthwise.

The section that divides the vessel into these two equal parts is called the diametrical section.

As these parts are not only equal, but as care is taken to load them equally, the centre of gravity of the whole vessel must fall in the diametrical section.

When a vessel floats upon the water, if we suppose a plane to pass through the water line, it will separate the part immersed from the remainder of the vessel. The part below this plane is called the *Hollow* of the vessel, and its centre of gravity the *Centre of the Hollow.*

457. There are two conditions essential to the equilibrium of a vessel floating upon a fluid, viz.

(1) The immersed part must be equal in volume to a mass of the fluid whose weight is equal to that of the vessel.

(2) The centre of gravity and the centre of the hollow must be in the same vertical line. This line is called the *Vertical Axis* of the vessel.

The centre of the hollow is always below the water line, but the centre of gravity may be either above or below it. In large vessels, particularly in ships of war, it is above the water line, in canal boats, &c., it is below. EULER, Book I.

458. Were the parts of vessels firmly fastened together, and the materials inflexible, or if their figure were such that the pressure of the water upwards, and the action of the weight of the vessel downwards, were not only in general equilibrio, but also in equilibrio at every point on the bottom of the vessel, the above conditions would be all that need be considered. But vessels are neither perfectly well fastened, nor are their materials inflexible; their shape, too, is such, that the equilibrium between the two pressures does not take place at every point.

If we conceive the vessel divided into two parts, by a transverse section passing through the vertical axis, and if we find the centre of gravity of each of these parts, and the centres of gravity of the corresponding portions of the hollow, the pressure of the water and the weight of these parts are in equilibrio only when the centres are situated in the same vertical line. In all other positions of these points, one or other of the forces acts to bend or change the figure of the vessel.

In ships of the usual figure, the lines of direction of the centres of gravity of the two parts fall farther from the transverse section than the lines of direction of the centres of the corresponding portions of the hollow. The weight has therefore an action to depress the vessel at each extremity. This change of shape is called *hogging*. Such is the intensity of this force, that all vessels of any considerable burthen have their shapes changed in some degree at the moment of launching; and it acts continually while the vessel lasts. To lessen the effect of this cause, Seppings has proposed to substitute diagonal trusses in place of the ceiling plank of vessels; and his plan has been found effectual in several ships built under his direction for the British navy. See QUARTERLY REVIEW, for 1812.

459. When a vessel, by the action of the wind, or of any other cause, becomes inclined, one of the three following consequences must happen:

(1) The vessel may remain in this inclined state, in which case we say the equilibrium is insensible.

(2) The vessel re-establishes itself by its own efforts in its original situation, when the equilibrium will be permanent.

(3) The vessel, after being inclined, is entirely overturned; this equilibrium is called unstable or tottering.

Neither the first nor the last of these ought to take place in a vessel; the second is absolutely necessary to insure stability.

460. When a vessel inclines, the position of the centre of gravity remains unaltered, but the centre of the hollow changes its place; if it change its position so far that the line of direction of the centre of gravity falls farther from the vertical axis than the vertical line drawn through the centre of the hollow, the equilibrium will be permanent, and the vessel will restore itself to its original position; if the line of direction of the centre of gravity falls without the vertical drawn through the centre of the hollow, the equilibrium is destroyed.

Where the external force acts to depress either extremity of the vessel, and the vessel tends to restore itself to an upright position, the motion is called Pitching; where the inclination is to either side, the motion is called Rolling.

461. The figure which the section of a vessel ought to have, in order first to resist an effort to incline it, and then restore itself, has occupied the attention of analysts. It appears from their researches, that if the sections in planes parallel to the direction of the disturbing forces be all circular, and have their centres in a line passing through the centre of those forces, the stability will be the greatest possible.

This is also confirmed by experiment, for a quarter of the rhind of a cocoa-nut, or of the skin of an orange, if thrown into the water, will float with its extremities elevated above the fluid, and resist the effort of the winds and waves to overturn it, or depress its extremities.

From a consideration of this circumstance, Mr. Greathead was led to the invention of his life boat: for an account of which, see *Philosophical Magazine.*

462. The stability of a vessel will also depend upon the proportion between its depth and breadth, upon the position of the centre of gravity, and of the centre of the disturbing forces.

When the vessel is propelled by sails, the last named point is called the *Centre Velique.*

In canal boats, vessels propelled by oars, &c., the centre of gravity is below the surface of the water; in many merchant ships, and in all vessels of war, the centre of gravity is above the water line. In the latter case, the breadth of the vessel can never be safely less than half her draught of water. This is of great practical importance, for many builders consider, that by increasing the depth of a vessel her stability is augmented, when the reverse is in fact the case.

463. As long as a body immersed in a fluid continues at rest, all points of its surface sustain equal pressures; but as soon as it begins to move, it is acted upon by another force that is directly opposed to its motion; this is the resistance of the water.

As from theory it would appear, that the same resistance will take place whether a body move in a fluid with a determinate velocity, or the fluid move and strike it with the same, the investigation of the law which obtains in the one case, will hold good in the other. The following theorem is therefore applicable to the motion of ships, as well as to the action of their rudders and sails.

464. If a fluid in motion impinge against a plane, the force with which it strikes will be proportioned to the specific gravity of the fluid, to the area of the plane, to the square of the velocity of the fluid, and the square of the sine of the angle at which the plane is inclined to the direction of the course of the fluid.

Or, if f represent the force of the fluid, s its specific gravity, a^2 the area of the plane, v the velocity of the fluid, and o the angle of obliquity:

$$f = s\, a^2\, v^2 \sin.^2 o$$

This theorem may be submitted to the test of experiment, by making use of a vessel of the form of a parallelopiped, and applying to it wedges of equal bases and unequal heights; and it will be found among them to be nearly if not exactly true.

When a vessel of the figure of a parallelopiped moves through water, the fluid, in consequence of its viscidity, does not close immediately behind it, and thus a *non-pressure* takes place. To obviate this, a wedge may be applied to the stern as well as to the bow, and the resistance will be found to diminish.

In consequence also of its viscidity, the fluid opposes a greater resistance to a wedge with plane surfaces, than to one whose surfaces are curved. And the wedge at the stern is better supported if it be curved also. The best form for the curves at the bow is found to be convex outwards, and that for the stern concave. If these be joined by a regular sweeping line instead of a straight one, thus removing all the angles, the resistance will be still further diminished. We thus arrive at the species of curve that the water lines of vessels should be composed of, and we find figures of the same kind adopted by nature in the shape of fishes, &c.

465. The action of the wind upon the sail of a vessel is such as would propel it in a direction perpendicular to the sail, provided the vessel were equally resisted in every direction, or were of a circular shape. As a vessel is less resisted in the direction of its length than in any other, the true course can be perpendicular to the plane of the sail only when the latter is perpendicular to the plane of the keel, and in no other case.

We consider the course as being in the plane of the keel, and apply a correction according to circumstances, that is called the *Lee-way.*

The lee-way will decrease with the increase of the length of the vessel, of her depth, and of the angle that the wind makes with the plane of the keel, and with the sails; but according to ratios differing in every different vessel.

466. Vessels are either rigged with square or with fore and aft sails; in the former, the yards may be braced until the angle they make with the plane of the keel is reduced to 40°, or even a little less; in the latter, the sails hang in the plane of the keel,

but cannot be used to advantage until the action of the wind has changed their position, and they make an angle of at least 20° with the keel. The primitive position of square sails is at right angles to the keel; fore and aft sails may be used in the same position.

By considering the action of the wind upon these sails, the possibility of plying to windward may be seen; thus, as a square rigged vessel will receive the wind in its sails, and be propelled nearly in the plane of its keel when the apparent direction of the wind makes an angle with the keel no greater than 45°; two courses, upon different tacks, will bring the vessel to a point directly to windward of that whence it first set off.

We say the *apparent* direction of the wind, because, when a vessel is in motion, the direction of the wind, as observed on board of her, is different from the absolute direction.

Vessels with fore and aft sails can lie still nearer the wind, and therefore can ply to windward with greater advantage.

Of all vessels, the flying prow of the Ladrone Islands lies nearest to the wind; the periauguas used in the harbour of New-York also lie very close to the wind; but of all fore and aft rigged vessels, the American pilot boat appears to combine the greatest number of advantages.

467. Ships and all other vessels are found to sail best when nearly in an upright position. As the action of the wind in oblique courses, tends to make them heel towards the lee side, it is a matter of no small importance to make the inclination as little as possible.

The force exerted by the wind upon the sail in propelling the vessel is proportioned to the surface of the sail; but that exerted to incline the vessel, acts upon the arm of a lever, and is therefore proportioned not only to the surface of the sail, but to the length of the mast. Hence it is a good rule, when the quantity of sail is to be increased, to do it rather by increasing the length of the yards than that of the masts.

468. Masts should in general neither lean towards the bow nor the stern of a vessel. In the former case, they tend to de-

press and bury the bow in the water; in the latter, a part of the power is lost in unnecessarily raising the bow from the water.

The natural action of masts, even when perpendicular, is to depress the bow; hence vessels are generally built so as to have, when at rest, a greater draught of water at the bow than at the stern.

Masts leaning aft are, however, sometimes advantageous, when the vessel is sharp built; that part of the force exerted to raise the bow from the water is not then lost, but tends to make the motion of the vessel more rapid, particularly in high seas.

469. The Rudder is applied to vessels for the purpose of guiding them and changing their course; were the vessel a plane surface, placed vertically in the water, the rudder would act with the greatest force when it makes with the keel an angle of 54° 44′. EULER, book II.

In consequence of the wedge-like form of the after part of a vessel, an angle of from 45° to 48° is found to be the best in practice. EULER, ib.

In oblique courses, the action of the rudder is much more powerfully exerted to turn a vessel from than towards the wind; it is for this reason that the manœuvre of veering must often be resorted to instead of that of tacking.

470. The time in which vessels whose breadth and depth are equal take to perform the manœuvres of tacking and veering, is proportioned to their length.

This gives the limit in practice to the length of a vessel, which could not be too great, were we merely to consider the facility of passing through the water, and the diminution of the lee-way. The usual proportion of the breadth to the length of vessels navigating the ocean, is between 1 : 4 and 1 : 3.

471. When ships of a great draught of water are to be transported over shoals, they are fitted to a machine that is called the *Camel*. This consists of two large hollow cases of timber, one side of each of which is of such a form as to apply itself closely to the convexity of a ship's figure. These cases are brought and sunk, by opening plugs, one on each side of the vessel to be

lightened; the plugs are then shut, and the water pumped out, the cases rise, embrace the ship, and lift it up with them.

These machines are principally used in Holland and in Russia.

OF THE STEAM ENGINE.

472. The Steam Engine is a machine where the expansive force of the elastic vapour of boiling water is applied, as a moving power to overcome any resistance whatsoever. It was originally a machine of little power, and was for a long time considered as applicable to but one purpose; but it has, since the time of Watt, become the most powerful of all those instruments by which natural agents are applied to the increase of human energy. We shall mention the several forms under which this engine appears to have existed, in a chronological order.

(1) Brancas, an Italian, appears to have been the first person who entertained the idea of using steam as a moving power; he proposed to apply the vapour issuing from the mouth of an Eolopyle to fans or sails placed on the circumference of a vertical wheel.

(2) The French claim the invention of the steam engine for their countryman Papin, known as the author of the digester. His project appears to have been similar to that of

(3) The Marquis of Worcester. In this engine, the steam was made to act upon the surface of water enclosed in a vessel by a valve, and to drive it up by a rising pipe, in the manner of a forcing pump.

(4) Captain Savary, who first executed a successful engine for the purpose of raising water, applied the steam on a principle composed of those of the common and forcing pump. In his engine, an ascending pipe reaches from the water of the reservoir to a chamber, the height of the upper part of which is less than 34 feet; if this be filled with steam, the water will rise into the vessel as soon as it is condensed, and will be retained there by the valve of the ascending pipe; if another pipe be adapted to the bottom of the chamber, furnished with a valve opening upwards, and if steam be admitted and made to act on the surface of the water, it will be drawn up through the second pipe to a height that will be determined by the elastic force of the steam.

In the machines of Savary, Papin, and Worcester, much heat is lost in consequence of the steam coming in contact with the cold water, and with the sides of the chamber, after it has been cooled down to the temperature of that fluid; they therefore use much fuel.

(5) In Newcomen and Cowley's engine, the steam was introduced beneath a piston fitted air tight into a cylinder; when the steam was condensed, the piston fell, being forced down by the pressure of the superincumbent air; when steam was again admitted beneath the piston, it rose, in consequence of its being more than counterbalanced by the weight of a pump rod attached to the opposite extremity of a beam vibrating upon a prop. Thus, a reciprocating motion was produced, and applied to the purpose of working a forcing pump.

In this machine, the atmosphere was evidently the moving power, and the steam merely applied to restore the equilibrium and produce a vacuum.

This engine is imperfect, in consequence of the great waste of steam; for this is introduced into a vessel exposed to alternations of heat and cold, in such a way that it can neither preserve the heat necessary to the full elasticity of the steam, nor become cold enough to produce its complete condensation.

For these reasons, the engine of Newcomen, in its best state, never acquired a power due to a pressure of more than 7⅓ pounds per square inch upon its piston; whereas the whole pressure of the atmosphere is double that quantity. The causes of the imperfect performance of these machines depend on the doctrine of latent heat; and until this was discovered by Dr. Black, no adequate remedy could be applied.

(6) The celebrated James Watt, the friend and pupil of Dr. Black, applied the principles of the doctrine of latent heat to the improvement of the steam engine. This he did by adding a separate vessel for the condensation of the steam. He called this vessel the condenser, and the effect it produced was such as to raise the power of the atmospheric engine from 7½ to 12 pounds per cubic foot.

With this addition, with more perfect workmanship, and more regular motion, (the latter of which is secured by the use of a fly,) the steam engine is known by the name of the *Single* or Atmospheric Engine of Watt.

(7) The Atmospheric Engine is incapable of being applied to produce any other than a reciprocating motion. In order to obtain a circular motion by means of the steam engine, Watt introduced steam both above and below the piston; in this way it is not only employed to form a vacuum, but becomes itself the moving power; it is not limited in its action to the limit of the pressure of the atmosphere, but may be extended to pressures of greater amount per square inch. These pressures will depend upon the heat of the steam employed.

(8) This part of Watt's plan was not extended by him farther than to steam of a temperature a little higher than that of boiling water. It being discovered that the elasticity of steam increased with the squares of the increments of temperature, steam is now employed at higher temperatures, and with much greater expansive powers. Engines where steam of this character is employed, are called high pressure engines. In a high pressure engine there is no condenser, but the steam is permitted to escape into the air. The steam is admitted alternately above and below the piston, and the action is due to the difference of the expansive energy of the high steam on the one side, and that which is issuing into the atmosphere and losing its temperature on the other.

Although the elasticity of steam increases with the square of the increase of temperature, this is not attended with the advantage it might appear at first sight to possess; for it seems, from late experiments, that the consumption of fuel is proportioned to the elasticity of the steam.

In the old atmospheric engine, the rectilineal reciprocating motion of the piston rod was adapted to the circular motion of the end of the beam, by means of chains; these applied themselves to successive portions of a circular arc whose radius was the distance of the extremity of the beam from its centre of motion. The same contrivance was adopted at the other end of the beam, to change the reciprocating circular motion communicated to it into the rectilineal motion, suited for working the pump rod.

In Watt's first atmospheric engines, the same plan was continued; when however he planned the double engine, where the rods require to be forced down as well as drawn up, he contrived an apparatus, by which a change might be effected, and the force of the ascending and descending piston made effective to produce reciprocating circular motions. This contrivance goes by the name of the Parallel Motion.

The principle in its simplest form is as follows, viz.

473. If two of the adjacent angles of a parallelogram be made to describe concentric circles, so that the side between them passes through their centre, and one of the remaining angles another circle having its convexity opposed to that of the two former; then the fourth angle of the parallelogram describes a line that does not differ sensibly from a straight line. PLAYFAIR, § 355. PRONY, § 1478. REES, article *Parallel Motion.*

474. To convert the reciprocating motions into continuous circular motions, Watt invented several ingenious contrivances acting on principles similar to those of a crank. The earliest of these was known by the name of the Sun-Planet wheel.

475. In the old atmospheric engines the valves were opened and shut by hand; after a time rack work was applied to this purpose and worked by the machine; this is still employed in some of the late engines. A more neat contrivance for this purpose is the eccentric motion; of this, there are two kinds, one by Murray, of Leeds, the other by Watt. See REES, article *Steam Engine.*

476. Most of the valves that are applicable to pumps, (see § 435,) have, at different periods, been adapted to the steam engine. There is also another, that appears preferable to any of them; it is known by the name of the *Slide Valve.*

477. To regulate the admission of steam into the cylinder, a Governor is employed; the production of steam is itself regulated by a self-acting damper, that increases or diminishes the draught of the chimney of the furnace.

478. In converting a reciprocating rectilineal into a continuous circular motion, by means of the working beam and parallel motions, a small portion of the force is lost, in consequence of obliquity of action. More is lost in consequence of the irregularity of the motion of the piston; the velocity of this is continually accelerated during the first half of its descent, continually retarded during the second, and is for an instant at rest; it then returns in the opposite direction, with a motion that is for a time accelerated, reaches its maximum, and is again retarded.

For this, no adequate remedy was discovered by Watt, for the best workmanship and the heaviest fly will not entirely overcome the defect. It has however been met, and fully corrected by Francis Ogden, an American engineer, to whom Watt paid the compliment of erecting, for his own purposes, an engine on this construction.

479. In Mr. Ogden's engine, there are two cylinders that are so arranged, that the motion of one shall be at its maximum when that of the other is null. The velocity of the impelled point of the machine is therefore very nearly uniform.

480. Besides the original purpose of working pumps, and thus drawing water from a great depth, steam engines are now employed to propel machinery of every possible kind; to impel boats, and to draw carriages upon rail-ways.

It is also said that they have applied them of late in England, to draw carriages upon common roads. An attempt of the same kind has likewise been made in Pennsylvania.

The earliest idea of the possibility of propelling boats by steam, seems to have originated with Savary; he even mentions the method of paddle wheels as the one he would propose to use. No practical result however followed from his theory, nor could his engine have been used for such a purpose. Watt has intimated in conversation, that he had at one time entertained views of attempting to apply steam to navigation, but was prevented by the pressure of other business connected with his engine.

Steam-boats appear to have been actually constructed and set in motion by Miller, of Dalswinton, and Symington in Great-Britain It is clear, however, that they could have promised but little, from their being considered at the time as futile attempts, and being soon forgotten.

The first successful application of steam, to the purposes of navigation, was by Fulton. It is impossible, sufficiently, to appreciate the ingenuity by which he overcame all the unforeseen difficulties to which his novel project was exposed; these were such as, in the hands of a person of less genius, would have resulted in utter failure. Most of the expedients thus introduced by Fulton, in the course of his experiment, and which in fact constitute the merit of his invention, have been servilely copied by those who attempt to rob him of the honour.

There is but one person who has any right to divide any portion of the honour due to Fulton; this is Robert L. Stevens, who planned a boat that was in successful operation on the Hudson within a few weeks of that of Fulton.

It appears by a report made to the British Parliament that one Bell, a mill-wright, in the employ of Fulton, was the first who constructed a successful steam-boat in Great-Britain. It is attempted, without any good ground whatever, to claim for him the honour of invention.

When steam engines are employed to draw carriages along roads or rail-ways, no other engine than that of high pressure can be used. In this, the great weight of water in the condensing cistern may be omitted. High pressure engines have also been used in steam-boats, but these are here unsafe, in consequence of their being more liable to explosion than those where the temperature of the steam does not much exceed 212° of Fahrenheit.

481. In steam engines of similar construction, the effects are nearly as the quantity of fuel consumed.

It is computed that an engine of the best construction will raise 20,000 cubic feet of water to the height of 24 feet for every hundred weight of coal consumed. PLAYFAIR, § 355.

482. The power of steam engines is estimated by a conventional force, whose unit is the strength of a horse. This has been variously estimated in different authors. Watt considers the power of a horse as equal to raise 32,000 pounds one foot high per minute; one fourth of this he conceives to be employ-

ed in overcoming the friction of the machine itself. The power of an engine will depend on the expansive force of the steam, on the area of the piston, on the length of the stroke of the piston rod, and on the number it makes in a minute.

If a^2 represent the area of the piston in square inches, p the pressure of the steam per square inch, n the number of strokes per minute, and l the length of the stroke; the number of horse powers will be represented by the formula:

$$\frac{2\,a^2\,p\,n\,l}{32000}$$

OF WIND-MILLS.

483. A Wind-mill is a machine in which a current of air is applied to produce a rotary motion.

The winds that are applicable to this purpose are such as have a velocity of not more than 30 feet per second, nor less than 12. When the velocity falls below 12 feet per second, a current of wind is found insufficient to do any effective work; when the velocity exceeds 25 feet per second, it becomes necessary to lessen the sails of the mill, in order to prevent the wings from being broken; and when it exceeds 30 feet, no sail can safely be spread. HACHETTE, § 176.

A knowledge of the absolute force of the wind is of little importance in practice, all that is required is to determine by experiment the force transmitted by means of a wind-mill.

Wind-mills are either vertical or horizontal.

484. In the vertical Wind-mill, the sails are so disposed as to turn in a vertical plane around a horizontal axis. The number of sails is usually four; and their form that of a parallelogram.

485. If we consider the action of a single sail separately, it is evident that if it were set exactly in the plane of vertical rotation, the wind would act only to overturn the mill, and would not communicate any motion to the arbour of the mill; it is there-

fore necessary that the sail should be inclined to the vertical plane.

When a surface at rest is acted upon by a fluid in motion, the maximum of the force by which it would be carried in a direction perpendicular to the course of the fluid is attained when the plane makes with the direction of the current an angle of 54° 44′. When the surface has a motion around a fixed axis in the prolongation of one of its sides, this condition is found no longer to hold good, nor is there any constant angle at which every part of the sail ought to be inclined to the vertical plane. Those parts which are nearest to the axis are most inclined, and those most distant are less so; the greatest angle of inclination in practice is 25°, and it is gradually diminished to 0°.

In the vertical wind-mill, the pressure of the wind acts upon the sail during its whole course; all the four sails, therefore, concur in producing the effect.

Smeaton has laid down a number of maxims in relation to wind-mills; these may be found in the Philosophical Transactions, and in the separate editions of his works; they are also detailed at length by GREGORY, in his 2d volume, by HACHETTE, § 185, and in REES, article *Wind-mill.*

486. Horizontal wind-mills have been sometimes constructed, but they are found to be much less powerful than those which are vertical. Their effect appearing, by the experiments of Smeaton, to be no more than $\frac{1}{8}$ of that produced by a vertical wind-mill, whose sails are of equal dimensions.

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